Answers for NSSH exam paper 1 type of questions, based on the syllabus part 1

Section 1 Time lapses; speed time graph

1. (a) (24:00 – 18: 40) + 05:37 = 5:20 + 5:37 = 10:57 so duration is 10h 57min

Or with a time line: ←20 m→ ←5 h → ←5 h → ←37 m→ (18:40 19:00 24:00 05:00 05:37) total: 5h + 5h + 20 m + 37 m = 10h 57 m

(b) Average speed = distance ÷ (total time used) = 380 ÷ (10 + $^{57}/_{60}$) = 34.7 km/h

2. (a) A speed of 10 m/s is build up in 20 sec so this is an acceleration of 0.5 m/s^2 as well: $v = at$ so $a = \frac{10}{20} = 0.5\frac{\text{m}}{\text{s}^2}$.

(b) Deceleration is 1 m/s^2 so it takes 10 sec to come to halt. The total time is 30 + 10 = 40 s.

(c) Distance travelled = area under the graph = ½ height ×(sum of parallel lines) = ½ × 10 ×(40 + 10) = 250 m

3. (a) 15 16 – 00 34 = 14 76 – 00 34 = 14 42 Race started at 14:42. Time line: ←18 m→ ←16 m→ (14:42 15:00 15:16)

(b) Winner takes 34 minutes – 50.3 sec = 33 minutes and 60 sec – 50.3 = 33 minutes and 9.7 sec.

4. (a) (i) Speed increased from 0 to 27 m/s in 3 seconds so acceleration is 9 m/s^2 [As well: $v = at$; a is the gradient]

(ii) Area under the graph = area triangle = ½ × base × height = ½ × 3 × 27 = ½ × 81 = 40.5 m.

(b) After 15 seconds Peter is still 1200 – 124 = 1076 m above the ground.

So 1076 ÷ 2 = 538 sec = 8 min 58 sec is the time used with a speed of 2 m/s.

Total time 8 min 58 sec + 15 sec = 9 m 13 s.

5. (a) Covered distance = area under the graph so $A = ½\, bh = 1800 = ½ \times 120 \times V \rightarrow V = 180 \div 6 =$ 30 m/s

(b) At 80 sec the speed is 15 m/s. The triangle to the right of 80 s has an area of ½bh = ½ × (120 – 80) × 15 = 300 m so distance travelled during the first 80 s is 1800 – 300 = 1500 m.

6. (a) (i) 2 hours

(ii) (40 ÷ x) hours

(b) Average speed= (total distance covered) ÷ (total time taken) = 100 ÷ (2 + (40 ÷ x) = $\dfrac{(100)\times\frac{x}{2}}{\left(2+\frac{40}{x}\right)\times\frac{x}{2}} = \dfrac{50x}{x+20}$

(c) $S = \dfrac{50x}{x+20} \xrightarrow{\times(x+20)} Sx + 20S = 50x$ or $Sx - 50x = -20S$ or $x(S-50) = -20S$ so that $x = \dfrac{-20S}{S-50}$

7. $126 \times \frac{1}{3600} = 0.035$m.

8. Same distance → area under the graph is the same!

For Alfred: Area trapezium = ½ $h(a + b)$ = ½ × 10(10 + 16) = 130 m.

For Benno: 130 = ½ bh or 130 = ½ × 16 × v → v = 130 ÷ 8 = 16.25 m/s

Section 2 Inequalities and number properties.

1. (a) $10 - 6.5x < 23 \xrightarrow{-10} -6.5x < 13 \xrightarrow{\div(-6.5)} x > -2$ [*The sign changes!*]

(b) Smallest integer is –1.

2. Irrational are: $\frac{1}{\sqrt{3}}$ and $\frac{1}{\left(\sqrt{3}\right)^3}$

3. (a) First $\underline{m = 3}$ (parallel lines), point A has coordinates $(-2, 0)$ so $B(4, 0)$ on the line $y = 3x + c$
substitute: $0 = 3 \times 4 + c \rightarrow \underline{c = -12}$
(b) $\underline{y < 3x + 6 \text{ and } y > 3x - 12}$ [$\underline{3x - 12 < y < 3x + 6}$ should be correct as well]

4. (a) In 2019 Jens will be 19 and Steven will be 23, both prime numbers.
(b) When Jens was 4 years then Steven was 8 years old this was in 2004.

5. If $y < -2$ then $-1 < y^{-1} < 0$; $y^0 = 1$; $y^2 > 1$ and $y^3 < -1$ so the correct sequence is: y^3 ; y^{-1} ; y^0 lastly y^2.

6. (a) The distance between two apple trees is 2×3 m = 6 m. (Using Pythagoras)
So 600 m ÷ 6 = 100 spaces so there are <u>101 apple trees</u> needed.
(b) The orange trees are one less: <u>100 orange trees.</u>

7. (a) $5 - \frac{2x}{3} > \frac{1}{2} + \frac{x}{4}$ multiply with 12: $60 - 8x > 6 + 3x$ or $-11x > -54$ or $x < 5$ so $x = 1, 2, 3$ or 4.

8. (a) $x = 3$ **(b)** $x = 2$ **(c)** $x = 3\frac{1}{3}$

<u>Section 3 Accuracy of measurements and significant figures.</u>

1. Upper bound for AB is 20.5 m; lower bound for CB is 2.45 m so the maximum possible length of the remaining piece AC is 20.5 – 2.45 = <u>18.05 m</u>

2. (a) 1.505 m
(b) **(i)** 1.835 – 1.505 = <u>0.33 m.</u>
(ii) <u>33 cm</u>

3. (a) Maximum time taken: 4h 15 min = 4.25h so latest possible arrival time is 05:55 + 4:15 = <u>10:10</u>
(b) speed × time = distance so max dist. = 102.5 × 4.25 = <u>435.625 km</u>

4. Dist. ÷ speed = time so $7 \times 10^{23} \div (3 \times 10^8 \times 24 \times 365) = 266362252663 = \underline{2.7 \times 10^{11}}$ <u>light years</u>
[The speed of light is not 3×10^8 km/h but 300 000 km/s = 1.1×10^9 km/s! As well: the universe is approximately 13.8 billion years old 13.8 billion years = 1.38×10^{10} so 2.7×10^{11} light years seems a rather big answer]

5. LB distance is 1875 km; UB speed is 105 km/h so minimum time is 1875 ÷ 105 = 17.8571h = <u>17h 51 min.</u>

6. (a) $1 \div 250\,000\,000 = \frac{1}{250000000} = \frac{1 \times 10}{250000000 \times 10} = \frac{10}{2.5 \times 10^9} = 4 \times 10^{-9}$ s.
(b) 250 000 000 operations in one second → $60 \times 250\,000\,000 = 15\,000\,000\,000 = \underline{1.5 \times 10^{10}}$ operations/m

7. (a) minimum length is 3 × 11.5 cm = <u>34.5 cm</u>
(b) minimum perimeter 8 × 11.5 cm = <u>92 cm</u>

8. Lower limit = $(21.5 \div 360) \times 2 \times 162.5 \times \pi$ = <u>61.0 cm.</u>

9. A car travels the circumference of the tire (not tyre) in one revolution, so $2 \times \pi \times 0.3 \times 7.5 \times 10^6 \times 7.5 \times 10^6$ m = <u>106029 km.</u>

10. $95 \times 5.97 \times 10^{24} = 567.15 \times 10^{24} = \underline{5.7 \times 10^{26}\text{ kg}}$

11. (a) 2.5 cm **(b)** $\tan^{-1}(10.5 \div 2.5) = \underline{76.6^\circ}$

12. Maximum distance = $\frac{52.5}{360} \times 2 \times \pi \times 20.5 + 41 = \underline{59.8\text{ m}}$

<u>Section 4 Variation</u>

1. (a) $E = c \times x^2$ substitute given values: $60 = 25c$ so $c = 2.4$ so the equation is $\underline{E = 2.4x^2}$.

(b) $135 = 2.4x^2 \rightarrow x = \sqrt{(135 \div 2.4)} = \underline{7.5\text{ cm}}$

2. (a) $F = \frac{c}{d^2}$ substitute F = 12 newtons and d = 1.5 meters: $12 = \frac{c}{1.5^2}$ so $c = 12 \times 1.5^2 = 27$ so eq. is $F = \frac{27}{d^2}$

(b) $F = 27 \div 0.6^2 = \underline{75\text{ Newton.}}$

3. $y \propto x^n$ so $y = cx^n$ ….① with c constant.

Use the second column: $\frac{1}{3} = c\,1^n$ so $c = \frac{1}{3}$ the relation is now: $y = \frac{1}{3}x^n$. Use the fourth column to find n::

$9 = \frac{1}{3} \times 3^n$ this means $\underline{n = 3}$ [Check: use column 3: is $2\frac{2}{3}$ equal to $\frac{1}{3} \times 2^3$ indeed $\frac{8}{3} = \frac{8}{3}$.]

4. (a) $T = k\,h$ with k a constant. Then $-6 = k \times 750$ so $k = -6/750 = -1/125 = -0.008$ so $T = -0.008 \times h$.

(b) (i) $-18 = -0.008 \times h$ so $h = 18 \div 0.008 = 2250$ m. So the mountain is 2250 + 2500 = <u>4750 m</u> high.

(ii) the temperature at sea level is $T = -0.008 \times (-2500) = \underline{20^\circ\text{C}}$

<u>Section 5 Polygons</u>

1. (a) angle $BOC = 360^\circ \div 5 = \underline{72^\circ}$ [Connect O with A,E and D; 5 congruent triangles make up the pentagon]

(b) angle $BEC = ½ \times 72^\circ = \underline{36^\circ}$ [Angle at the centre is double the angle at the circumference]

(c) angle $BDC = \underline{36^\circ}$ [$\angle BEC$ and $\angle BDC$ are both subtended by chord BC so they are equal,]

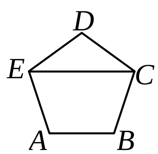
2. (a) A <u>square</u> has rotational symmetry of order 4.

(b) A <u>rectangle</u> has 4 right angles but only two lines of symmetry.

(c) A <u>kite</u> has exactly one line of symmetry.

3. Sum of all exterior angles is 360º so $3 \times 40^\circ + 2 \times 50^\circ + x \times 20^\circ = 360^\circ \rightarrow 20x = 360 - 220 \rightarrow \underline{x = 7}$

4. Draw the pentagon:



(a) exterior angle is $360 \div 5 = 72^\circ$ so angle $ABC = 180 - 72 = \underline{108^\circ}$

(b) ΔEDC is an isosceles triangle so angle $BCE = 108 - 36 = \underline{72^\circ}$

Section 6 Similar shapes

1.(a) 4.5 m ÷ 20 = 450 cm ÷ 20 = 22.5 cm.[Measurements in a model are small as compared to the real Measurements; all measurements are reduced by a factor 20.]

(b) Ratio lengths is 1 : 20 so the ratio between the volumes is $1^3 : 20^3$ = 1 : 8000 so the volume of the interior is 8000 times bigger → volume of the interior of the real car is 600 $cm^3 \times 8000 = 4800000\ cm^3 = 4.8\ m^3$

2. (a) The poster size is 6 × bigger than the post card [90 ÷ 15 = 6] so the church on the poster = 6 × 7.5 = 45 cm.

(b) Ratio lenghts is 1 : 6 so ratio area postcard : area poster = 1 : 36 → area sky postcard is 936 ÷ 36 = 26 cm^2

3. $V_A : V_B$ = 1080 : 320 = 27 : 8 → $L_A : L_B$ = 3 : 2 → $A_A : A_B$ = 9 : 4. So the area of the label of tin A is 90 cm^2

4. $V_S : V_B$ = 100 : 800 = 1 : 8 → $L_A : L_B = \sqrt[3]{1} : \sqrt[3]{8}$ = 1 : 2 → the height of the larger bowl is 10 cm.

5. $L_M : L_B$ = 1 : 20 → $A_M : A_B$ = 1 : 400 → area of the roof of the model = 300 ÷ 400 = 0.75 m^2

6. (a) 500 × 9 cm = 4500 cm = 45 m

(b) Scale is 1 : *m* so the distance have ratio 1 : *m* so areas are in the ratio 1 : m^2. It is given that the ratio of the Areas is *p* : *P* so 1 : $m^2 = p : P$ or $\frac{p}{P} = \frac{1}{m^2}$ apply cross multiplication: $pm^2 = P$ or $m^2 = \frac{P}{p}$ or $m = \sqrt{\frac{P}{p}}$

7. (a) ΔABC is similar with ΔAQP so 4 cm × *f* = 3 cm; *AB* and *AQ* are corresponding sides so $f = \frac{3}{4}$ that means $PQ = \frac{3}{4} \times 3.6$ = 2.7 cm.

(b) For similar shapes we have (length sh I) : (length sh II) = *a* : *b* then (area sh I) : (area sh II) = $a^2 : b^2$.
So we get area ΔAPQ : area $\Delta ABC = 3^2 : 4^2$ or area ΔAPQ : 5.6 = 9 : 16 so area $\Delta APQ = 5.6 \times \frac{9}{16}$ = 3.15 cm^2

Section 7 Finance & percentages

1. (a) 2400 ÷ 11.50 = £209

(b) Let the exchange rate in 2009 be £ *x*; then 0.7 × *x* = 11.50 so *x* = 11.5 ÷ 0.7 = £16.43

2. 110% coresponds with 6.16 m→ Susan jumped (6.16 ÷ 110) ×100 = 5.6 m

3. (a) 1.16 × 470 = 545 mm.

(b) Let *x* be the rain fall in 2002 then 1.29 × *x* = 470 → *x* = 470 ÷ 1.29 = 364 mm

4. (a) Let the cost price be *x* then 1.5 × *x* = 225000 → *x* = 225000 ÷ 1.5 = N$ 150 000

(b) 0.15 × 225000 = N$ 33750

5. (a) $P_t = P_0(1 + \frac{r}{100})^t$ so $P_t = 2000(1 + 0.12)^5 = 2000 \times 1.12^5$ = N$ 3525 **(b)** $\frac{3525}{2000} \times 100 =$ 176%

6. (a) 120 × 325 = 39000g = 39 kg

(b) 300 ÷ 800 = 0.375 ℓ = 375 ml.

(c) $\frac{7}{20} \times 160$ = 56 kg of fruit.

(d) In 2012 fuel will cost 0.45 × 70000 × 1.04 = N$ 32760

Other materials will cost: 0.55 × 70000 × 1.1 = N$ 42350

Total cost in 2012 is: + N$ 75110 so the cost went up by N$ 5110.

As percentage: $\frac{5110}{70000} \times 100\%$ = 7.3%.

7. (a) N$8.10 in 3 months means 4 × 8.10 = N$32.40 per year. 32.49 ÷ 360 × 100 = 9% p.a.

(b) $20000(1 + 0.05)^3 = 20000 \times 1.05^3$ = N$23152.50

8. (a) 0.08 × 483 = N$38.64 so rounded off the tip is N$38

(b) Let x be the cost of the meal then 0.15 × x **+** x = 483 so x = 483/1.15 = N$420.

9. $\frac{1}{12}$ ($\frac{1}{3} - \frac{1}{4} = \frac{4-3}{12}$)of the cost of the car is N$5000. So the cost of the car is 12 × 5000 = N$60 000

Section 8 Linear programming

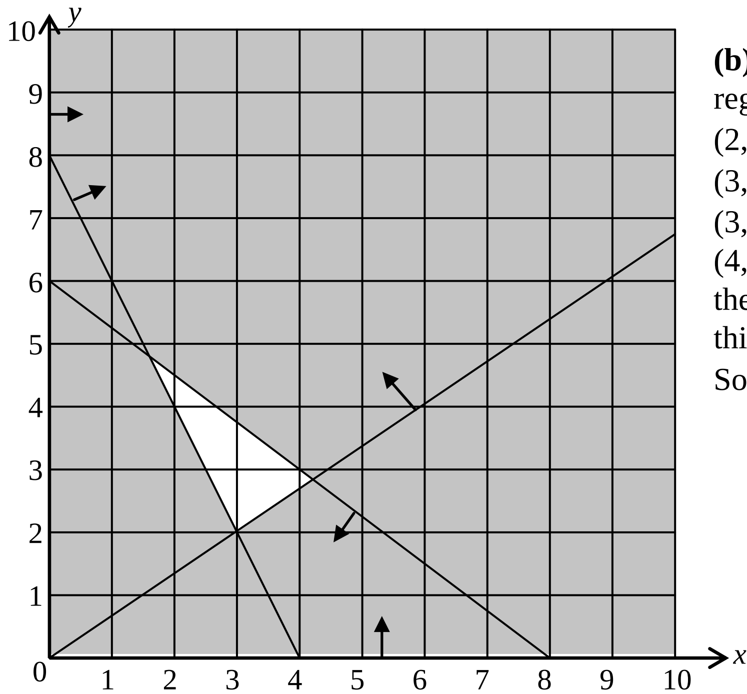
1. ① $x \le 4$ ② $y \ge 0$ ③ $y \ge -x + 3$ and ④ $y < x + 1$

2. (a) ① → $x + y \le 8$; ② → $x > 2$ ③ → $y < -0.25x + 3$

(b) Profit for A: 3 × N$1000 + 4.2 × N$500 = 3000 + 2100 = N$5100

Profit for B: 6 × N$1000 + 2 × N$500 = 6000 + 1000 = N$7000. So option B is giving more profit.

3. (a)



(b) Try all the points inside the region:

(2, 4) → 2

(3, 3) → 0

(3, 2) → –1

(4,3) is outside the region check the equation $4y + 3x = 25$ for this point.

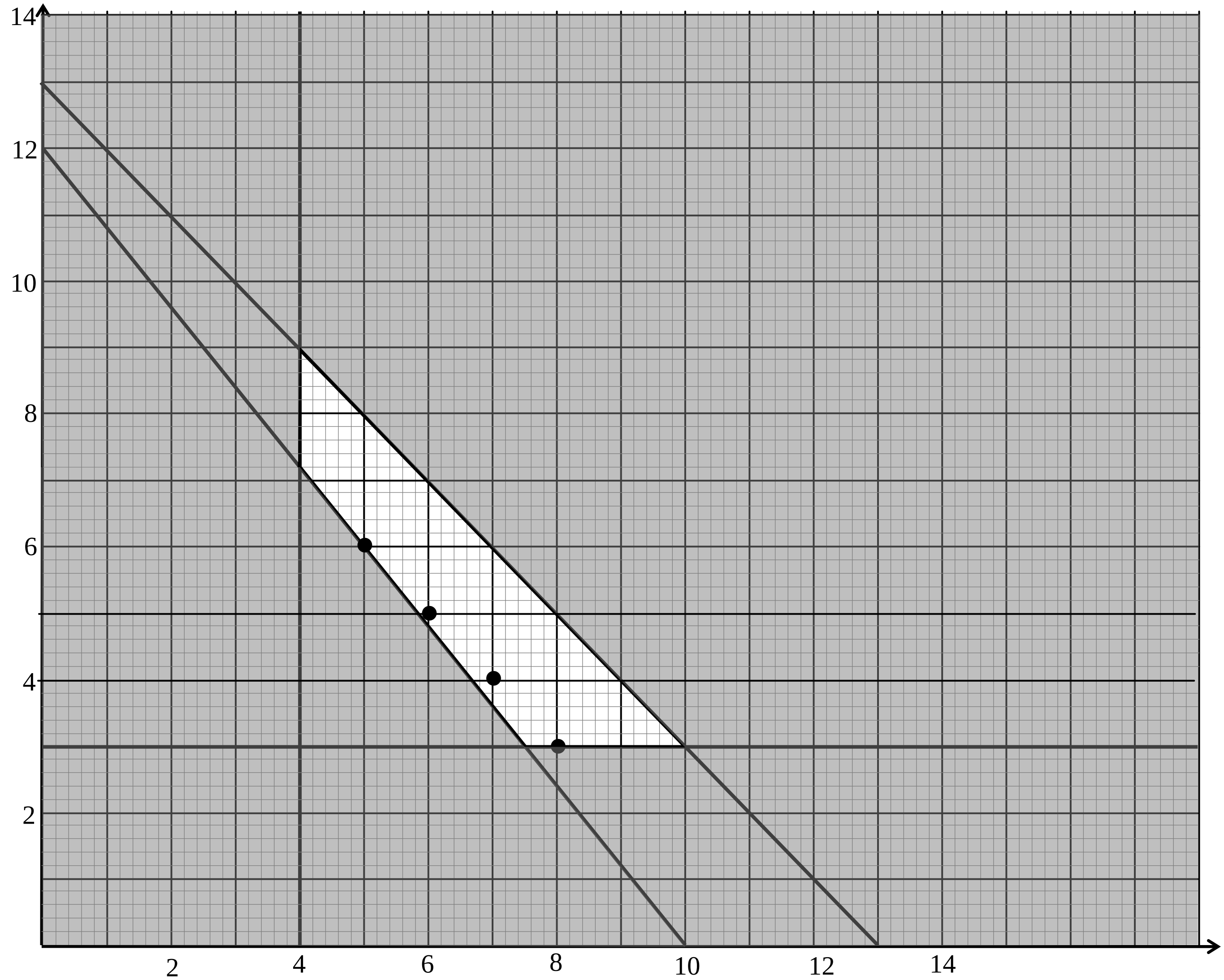
So the least value is –1.

4. (a) Valid is: $24x + 20y \geqslant 240$ divide by 4: $6x + 5y \geqslant 60$

(b) $x + y \le 13$

(c) $x \geqslant 4$ and $y \geqslant 3$

(d) See next page



(e) (i) the dots are acceptable solutions; for all those dots $x + y = 11$ so least number of people required: 11.

(ii) (10, 3) gives $10 \times 24 + 3 \times 20 = 240 + 60 = 300$ kg. (4, 9) gives $4 \times 24 + 9 \times 20 = 276$ kg.

So 300 kg is the greatest amount of soil which can be carried.

Section 9 Trigonometry and bearings

1. (a) Use the cosine rule: $AC^2 = 45^2 + 67^2 - 2 \times 45 \times 67 \times \cos 115.2^\circ =$ <u>95.3 km</u>

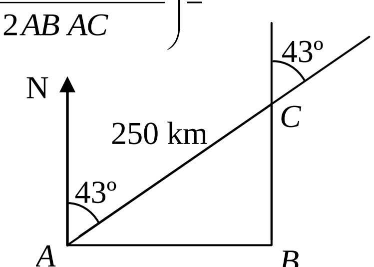
(b) Use sine rule to find $\angle BCA$: $\frac{\sin B\hat{C}A}{45} = \frac{\sin A\hat{B}C}{95.2966}$ so $B\hat{C}A = \sin^{-1}\left(\frac{45\sin 115.2^\circ}{95.2966}\right) =$ <u>25.3º</u>

So the bearing of A from C is 25º + 270º = <u>295º.</u>

2. (a) $\tan \angle ABP = \frac{AP}{AB}$ or $\tan 4^\circ = \frac{AP}{280}$ so height radio mast = $AP = 280 \times \tan 4^\circ =$ <u>19.6 m</u>

(b) Use cosine rule: $BC^2 = AB^2 + AC^2 - 2AB\,AC \cos CAB$ so $\angle CAB = \cos^{-1}\left(\frac{BC^2 - AB^2 - AC^2}{-2AB\,AC}\right) =$

$\text{Cos}^{-1}\left(\frac{430^2 - 280^2 - 260^2}{-2 \times 280 \times 260}\right) =$ <u>105.5º</u>



(c) Area $\triangle ABC = \frac{1}{2}\, AC\, AB \sin \angle CAB = \frac{1}{2} \times 260 \times 280 \sin 105.5^\circ =$ <u>3.51 ha</u>

3. (a) Extend BC and AC, another angle of 43º is then created. [As well alternating angles]
Bearing of A from C is now 180º + 43º = 223º.

(b) $\angle CAB$ = 90º – 43º = 47º.

In ΔABC, $\cos \angle CAB = \dfrac{AB}{AC}$ so that AB = 250 cos 47º = <u>170 km.</u>

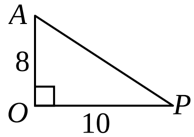
4. (a) Area $\Delta DEF = \frac{1}{2} ED.DF \sin \angle EDF = \frac{1}{2} \times 120 \times 70 \times \sin 130^\circ$ = <u>3.22×10^3 m^2</u>.

(b) Use the cosine rule: $EF^2 = ED^2 + DF^2 - 2ED.DF \cos\angle EDF$ or

$EF = \sqrt{120^2 + 70^2 - 2\times 120 \times 70 \cos 130^o}$ = <u>173 m</u> indeed.

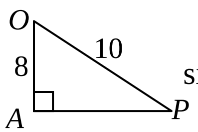
(c) Use the sine rule: $\dfrac{\sin D}{EF} = \dfrac{\sin E}{DF}$ or $\dfrac{\sin 130}{173} = \dfrac{\sin DEF}{70}$ so that angle DEF = $\sin^{-1}\left(\dfrac{70 \times \sin 130}{173}\right)$ = <u>18.0º</u>

5. (a) Draw the triangle:



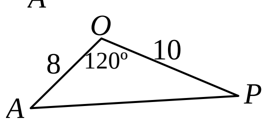
$AP^2 = 8^2 + 10^2 = 164 \rightarrow AP = \sqrt{164} = 12.8$ cm

(b)



$\sin APO = \dfrac{OA}{OP} = 0.8 \rightarrow \angle APO = 53.1^\circ$

(c)



$AP^2 = AO^2 + OP^2 - 2AO\,OP\cos 120$ = 64 + 100 – 160 (–0.5) → $AP = \sqrt{224} = 15.0$

6. (a) The bearing of C from B is 180 – 40 – 100 = 040º

(b) With the cosine rule: $AC^2 = AB^2 + BC^2 - 2AB \times AC \cos 100^\circ$.

$AC = \sqrt{(100 + 196 - 2 \times 10 \times 14 \cos 100^\circ)}$ = <u>18.6 km</u>

(c) $AB + BC$ = 24 km at a speed of 18 km/h this means Jens took 1hour 20 min.

Sven ran 18. 6.. km in $1\frac{1}{3}$ hour = $\frac{4}{3}$ hour so his speed was $\frac{3}{4} \times$ 18.6.. = 13.9 km/h

7. (a) 360 – 115 = <u>245°</u>

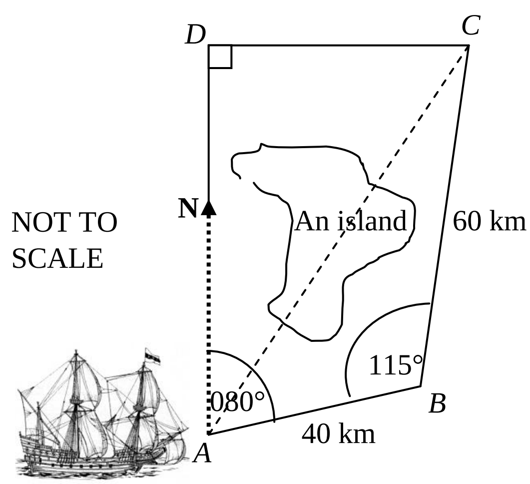
(b) Use the cosine rule: $AC^2 = AB^2 + BC^2 - 2AB\,BC \cos\angle B$ substitute the values:

$AC = \sqrt{[40^2 + 60^2 - 2 \times 40 \times 60 \cos 115]}$ = <u>85.0 km</u>

(c) Use the sine rule: $\dfrac{\sin\angle B}{AC} = \dfrac{\sin BAC}{BC}$ substitute: $\dfrac{\sin 115}{85.02} = \dfrac{\sin BAC}{60}$ so $\angle BAC = \sin^{-1}\left[\dfrac{60 \sin 115}{85.02}\right]$ = <u>39.8°</u>

(d) In ΔADC; $\angle CAD$ = 80 – 39.8 = 40.2°

So $\sin CAD = \dfrac{DC}{AC}$ or DC = 85.02 ×sin40.2° = <u>54.9 km</u>



Section 10 Transformations

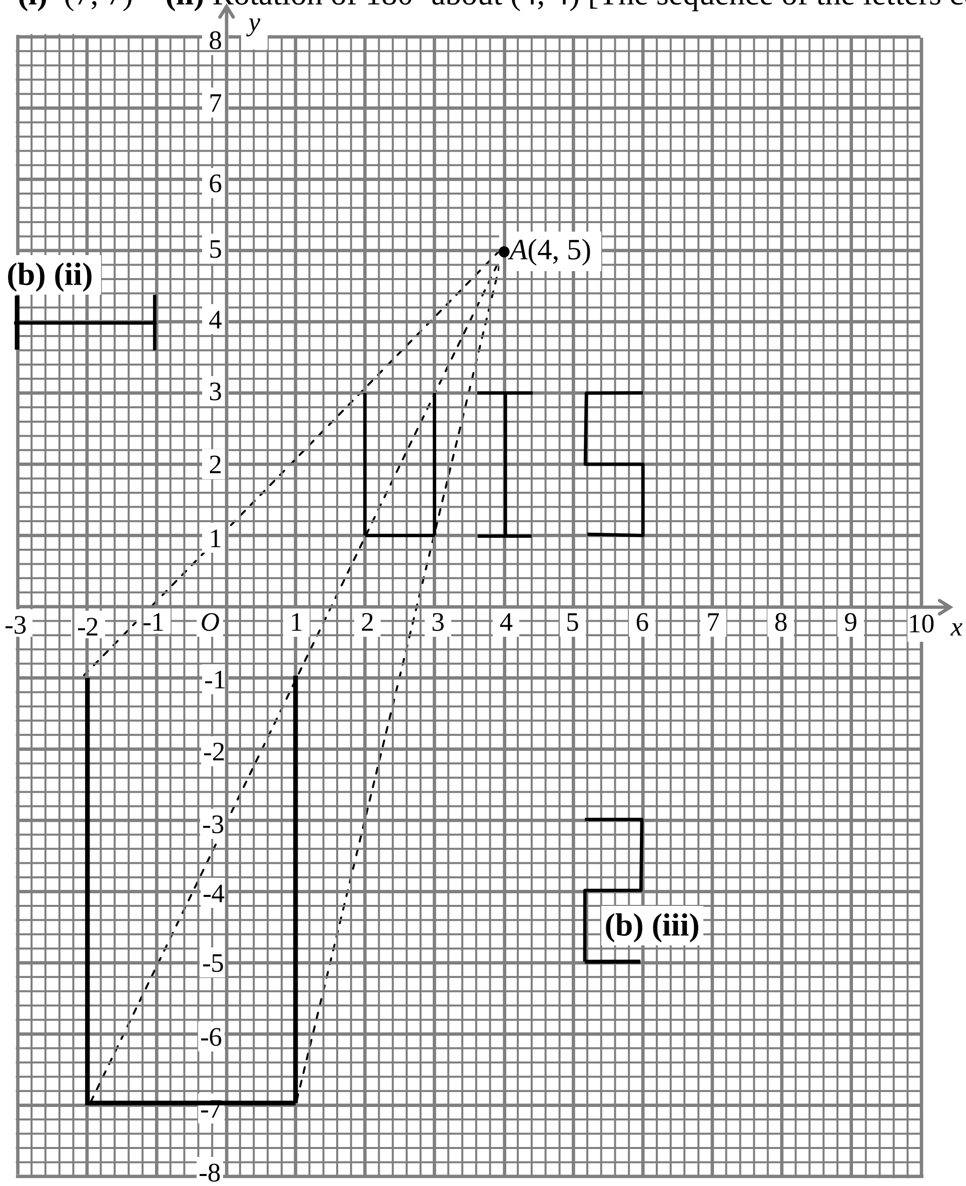
1. (a) Image of A is: (15, 2)

(b) Image of C is: (6, −5)

2. (a) **(i)** P **(ii)** −3 **(iii)** 2 **(iv)** (1, 10) **(v)** 4 : 1

(b) **(i)** (7, 7) **(ii)** Rotation of 180º about (4, 4) [The sequence of the letters counts: P to R; Q to S and R to P]

3. (a)



4. (a) Image of A is (3, 4)

(b) Image of B is (6, 2)

(c) Image of C is (4, 3)

5. (a) triangle F; $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(b) triangle D; reflection in the line $x = 1$

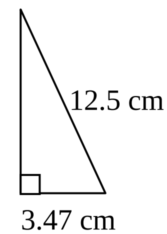
(c) triangle E; rotation about (2, –½)

6. (a) **(i)** translation $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ **(ii)** Rotation of 180° about the origin. [or enlargement centre origin factor −1]

(b) **(i)** (0, 4) or point A. **(ii)** B(1, 1)

7. (a) Image is: (−3, −2)
(b) Image is: (−2, 3)
(c) Image is: (0, −1)

Section 11 Mensuration



1. (a) The arc length of the sector = $\frac{100}{360} \times 2 \times \pi \times r$ = $\frac{5}{18} \times 2 \times \pi \times 12.5$ = 21.8 cm

(b) The circumference of the base of the cone is now 21.8 cm.
Name the radius of the base r_b , valid is then: 21.8 = $2\pi r_b$ so r_b = 21.8 ÷ 2π = 3.47 cm

(c) Use Pythagoras: $h = \sqrt{(12.5^2 - 3.47^2)}$ = 12.0 cm [12 cm is incorrect; it has to be rounded off]

2. (a) Volume cube = $s^3 = 6^3$ = 216 cm^3

(b) Apply formula: $V_{cyl} = \pi r^2 h$ = 216 cm^3 substitute h : $14\pi r^2 = 216$ so $r = \sqrt{216 \div 14\pi}$ = 2.22 cm

(c) 2 cm/h means 2 cm in 1 hour, so in 3600 sec; 0.02 m burns or in 1 sec: 0.02 ÷ 3600= 5.56×10^{-6} m/s

3. 1 nautical mile = $\frac{angle}{360} \times 2\pi r = \frac{1/60}{360} \times \pi \times d = \frac{\pi \times 12756}{360 \times 60}$ = 1860 m [Rounded off in 3 s.f.]

4. (a) (i) Use cosine rule: $AB^2 = BC^2 + AC^2 - 2\ BC.AC \cos BCA$ substitute the values given:

$$AB = (5^2 + 5.4^2 - 2 \times 5 \times 5.4 \times \cos 116)^{½} = 8.82 \text{ m}$$

(ii) CD can be calculated by the sine rule in ΔACD.

ΔACD has exterior angle 116º so $\angle D$ = 116 – 43 = 73º.

$$\frac{\sin ADC}{AC} = \frac{\sin CAD}{CD} \text{ so } CD = \frac{AC\ \sin CAD}{\sin\ ADC} = \frac{5 \times \sin 43}{\sin 73} = 3.57 \text{ m}$$

(b) (i) Parallel to CD → it will have half of the length of CD so ½ × 3.57 = 1.78 m

(ii) Perpendicular to AD → the trig ratio's can be applied:

$\sin \angle CAD = \frac{\text{length of the new beam } (x)}{\text{halve of } AC} = \frac{x}{2.5}$ so the length of the new beam x = 2.5 sin 43º = 1.70 m

5. (a) (i) V = 12 × 20 × 20 = 4800 cm^3

(ii) New volume is 3000 = 20 × 20 × h or h = 3000 ÷ 400 = 7.5 cm

(b) (i) Volume = area front surface × depth = area ΔBDC × 20 = ½ × 20 × 12 × 20 = 2400 cm^3

(ii) With Pythagoras $BD = \sqrt{BC^2 + DC^2} = \sqrt{20^2 + 12^2}$ = 23.3 cm

The vertical height of B above the table can now be calculated through the area:
Area ΔBDC = ½ × BC × DC = 120 cm^2 but the area is as well ½ BD × height.

Requested height = $\frac{120}{\frac{1}{2} \times 23.3}$ = 10.3 cm

6. (a) After folding the net to a pyramid and slicing it half you to see:
Use Pythagoras: $h = \sqrt{(25^2 - 7^2)}$ = 24 cm

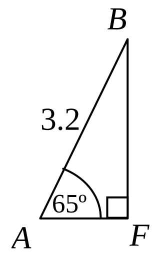
(b) Volume pyramid = $\frac{1}{3}$ × height × [area base] = $\frac{1}{3}$ × 24 × 14^2 = 1568 cm^3

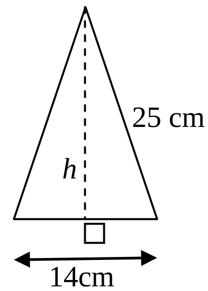
7. (a) (i) Drop a perpendicular from B to AE:

Cos $\angle A = AF/\ AB$ so $AF = 3.2 \times \cos 65^{\circ} = 1.35$ cm.

$BX = ½(8\text{ cm} - 2 \times 1.35) = \underline{2.648\text{ cm}}$

(ii) In ΔBXC $\sin \angle BCX = \dfrac{BX}{BC} = \dfrac{2.648}{3}$ so $\angle BCX = 62.0^{\circ}$





(b) Use the cosine rule in ΔABE: $BE = \sqrt{AB^2 + AE^2 - 2AB\,AE\cos BAE} = \sqrt{3.2^2 + 8^2 - 2 \times 3.2 \times 8 \cos 65} =$

$= \underline{7.25\text{ cm}}$

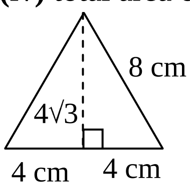
(c) (i) area of triangle $BCD = ½ \times BC \times CD \times \sin \angle BCD = ½ \times 3 \times 3 \times \sin 124^{\circ} = \underline{3.73\text{ cm}^2}$

(ii) area of trapezium $ABDE = ½$ height $(AE + BD) = ½\,(3.2 \sin 65^{\circ}) \times (8 + 2 \times 2.648) = \underline{19.3\text{ cm}^2}$

(iii) area of major sector $BND = \dfrac{360-124}{360}\pi r^2 = \underline{18.5\text{ cm}^2}$

(iv) total area of the cloud diagram $= 3.73 + 19.3 + 18.5 + \pi r^2 = 41.53 + \pi \times 1.6^2 = 49.6\text{ cm}^2$

8. (a)



Area is $½ \times b \times h = ½ \times 8 \times 4\sqrt{3} = 16\sqrt{3} \approx 27.7\text{ cm}^2$

(b) Circumference is 8 cm $\rightarrow 8 = 2\pi r \rightarrow r = 4 \div \pi = 1.27$ cm

(c) (i) Triangular pyramid; TSA $= 4 \times 16\sqrt{3} = 64\sqrt{3} \approx 111\text{ cm}^2$

(ii) Cylinder; $V = \pi r^2 h = \pi \times (4 \div \pi)^2 \times 8 = 128/\pi \approx 40.7\text{ cm}^3$

(iii) Cone; $h = \sqrt{(64 - [4 \div \pi]^2)} = 7.90$ cm.

9. (a) Top angle at in $\Delta ABO = 360^{\circ} \div 5 = 72^{\circ}$ so $\angle OAB = (180 - 72) \div 2 = \underline{54^{\circ}}$

(b) $\tan \angle OAB = \tan 54^{\circ} = OM/AM = OM/6$ so $OM = 6 \times \tan 54^{\circ} = \underline{8.26\text{ cm}}$

(c) Area pentagon $= 5 \times$ area $\Delta ABO = 5 \times ½ \times h \times b = 2.5 \times 8.26.. \times 12 = \underline{248\text{ cm}^2}$

(d) Area of the whole design = area 2.5 circles + area pentagon $= 2.5 \times \pi \times 6^2 + 248 = \underline{530\text{ cm}^2}$

10. (a) Use the sine rule: $\dfrac{\sin A_1B_1C}{AC} = \dfrac{\sin C}{A_1B_1}$ or $\dfrac{\sin A_1B_1C}{20} = \dfrac{\sin 36}{50}$ so $\sin A_1B_1C = 0.4 \sin 36$ so $\underline{A_1B_1C = 13.6^{\circ}}$

(b) Use the cosine rule: $B_2C^2 = A_2B_2^2 + A_2C^2 - 2\,A_2B_2 \times A_2C \times \cos A_2$ or

$B_2C = \sqrt{(50^2 + 20^2 - 2 \times 50 \times 20 \cos 125^{\circ})} = 63.6$ cm

(c) (i) arc length $= \dfrac{\angle A_3CT}{360^{\circ}} \times 2\pi r$ or 15 cm $= \dfrac{\angle A_3CT}{360^{\circ}} \times 2\pi \times 20$ so $\angle A_3CT = \dfrac{15 \times 360}{40\pi} = \underline{43.0^{\circ}}$

(ii) $B_3TC = 90^{\circ}$ because B_3T is a tangent to the circle. So $B_3C^2 = 20^2 + 35^2$ so $B_3C = 40.3$ cm

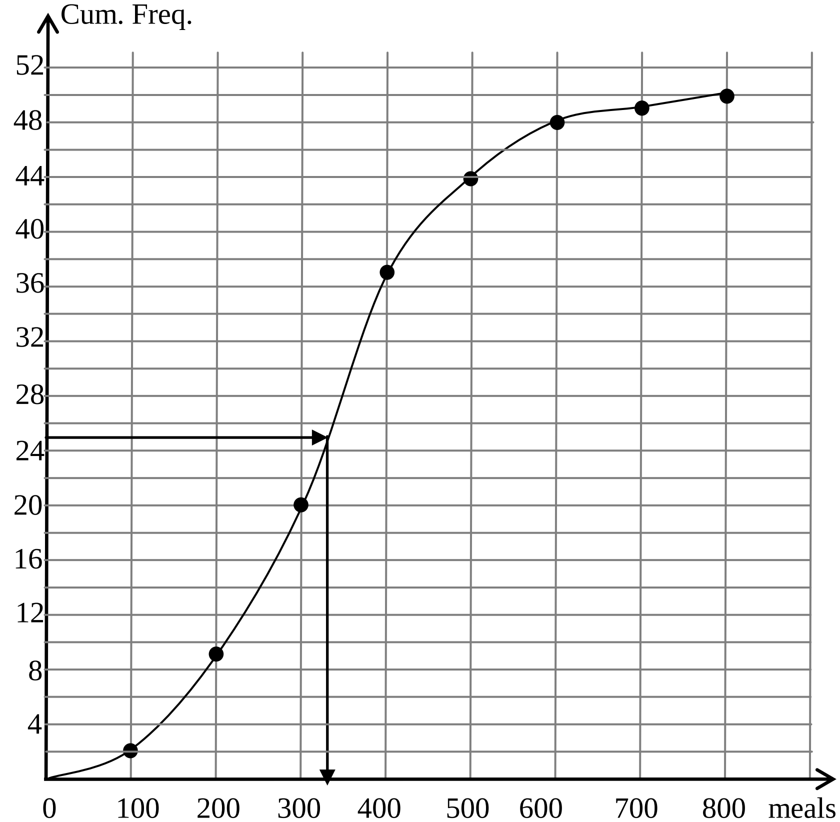
So that $B_3Q = 40.3 - 20 = \underline{20.3\text{ cm.}}$

Section 12 Statistics

1. (a) See graph to the right.

(b) Median is 325 meals.

(c) So the average number of meals is
$16600 \div 50 =$ 332 meals.



Number of meals (*M*) served per week.	**No of wks *f***	**Cum freq**	**Mid Point *m***	$f \times m$
$0 < M \le 100$	2	2	50	100
$100 < M \le 200$	7	9	150	1050
$200 < M \le 300$	11	20	250	2750
$300 < M \le 400$	17	37	350	5950
$400 < M \le 500$	7	44	450	3150
$500 < M \le 600$	4	48	550	2200
$600 < M \le 700$	1	49	650	650
$700 < M \le 800$	1	50	750	750
	50			16600

[Remark: the *x* coordinate is the class upper boundary and the *y* coordinate the acc. freq.]

2. (a) Remember: [Frequency density = frequency $\div$ class width] so from the first column we get:
$36 \div 5 = 7.2$ as frequency density. This results in a column with height 14.4 cm. We have to conclude that $2 \times$ the density gives the height of the column.
For the third column we get: $2 \times$ freq $\div$ class widt $= 2 \times 25 \div 10 =$ 5 cm

(b) [$36 \times 7.5 + 24 \times 12.5 + 25 \times 20 + 48 \times 32.5$] $\div 133 =$ 19.8 kg is the mean mass.

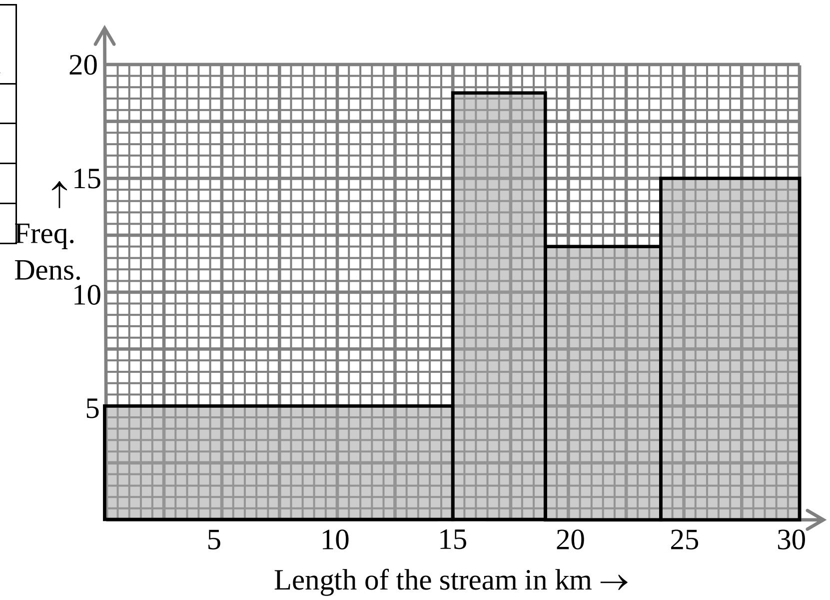
3. (a) If 3 km is the median then $x = 3 + 7 + 2 = 12$.

(b) [$12 \times 2 + 1 \times 3 + 3 \times 4 + 7 \times 5 + 2 \times 6$] $\div 25 = 86 \div 25 =$ 3.44 km

4. (a)

Length (*x* km)	Frequency	Density = freq ÷ width
$0 < x \le 15$	75	5
$15 < x \le 19$	75	18.75
$19 < x \le 24$	60	12
$24 < x \le 30$	90	15

(b) ➔ ➔



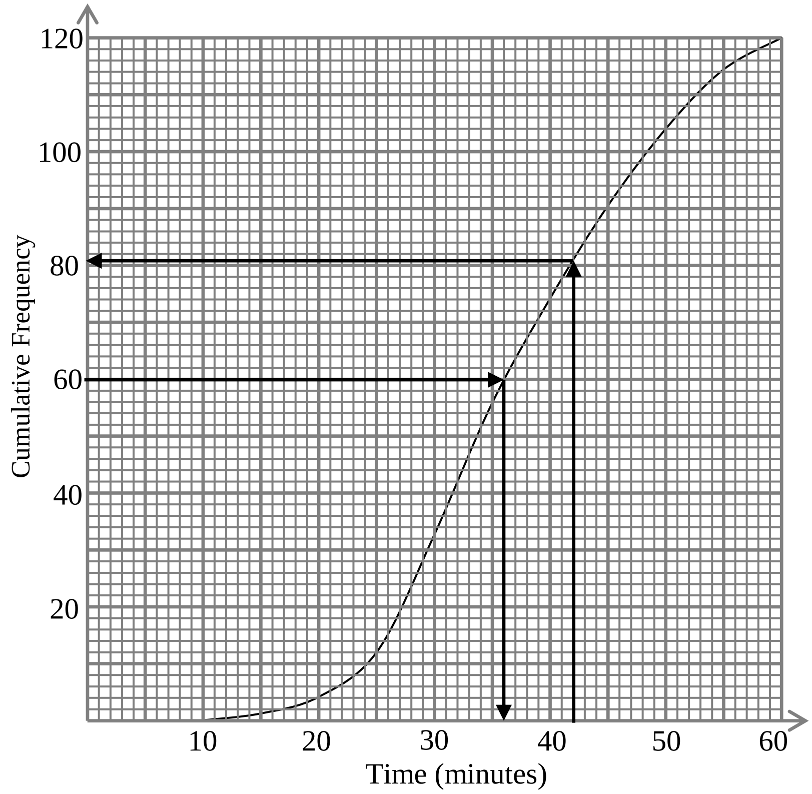
5. **(a) (i)** median time is 36 minutes

(ii) 81 students less than 42 min so

$\frac{120-81}{120} \times 100 = \underline{32.5\%}$

(b) From the graph (30, 32) is a point on the curve, so a = 32 – 4 = <u>28.</u>

b = 120 – (4 + 28 + 31 + 41) = <u>16.</u>



6. **(a)** p = 100 and q = 80 [by piling the different 'blocks' on top of each other.]

More official: Apply the rule: [Frequency density = frequency ÷ class width]

For the first class the height is 40 ÷ 5 = 8 so for the density 4 small blocks represent a density of 8.

For the third class: 10 = p ÷ 10 so <u>p = 100.</u>

For the fourth class: 4 = q ÷ 20 so <u>q = 80.</u>

(b) mean height = [40 × 152.5 + 80 × 157.5 + 100 × 165 + 80 × 180] ÷ 300 = <u>165.3 cm</u>

(c) The medain height falls in the class of $160 < h \leq 170$. [40 + 80 = 120 and 40 + 80 + 100 is more than 150 which is halve of 300]

7. **(a)** <u>2 years 11 months</u> [only age that appears twice]

(b) 3 years and 5 months [The measurement in the 'middle']

(c) Express all of the measurements in months: (41 + 45 + 35 + 40 + 52 + 46 + 35) ÷ 7 = 42 months = 3 years and 6 months.

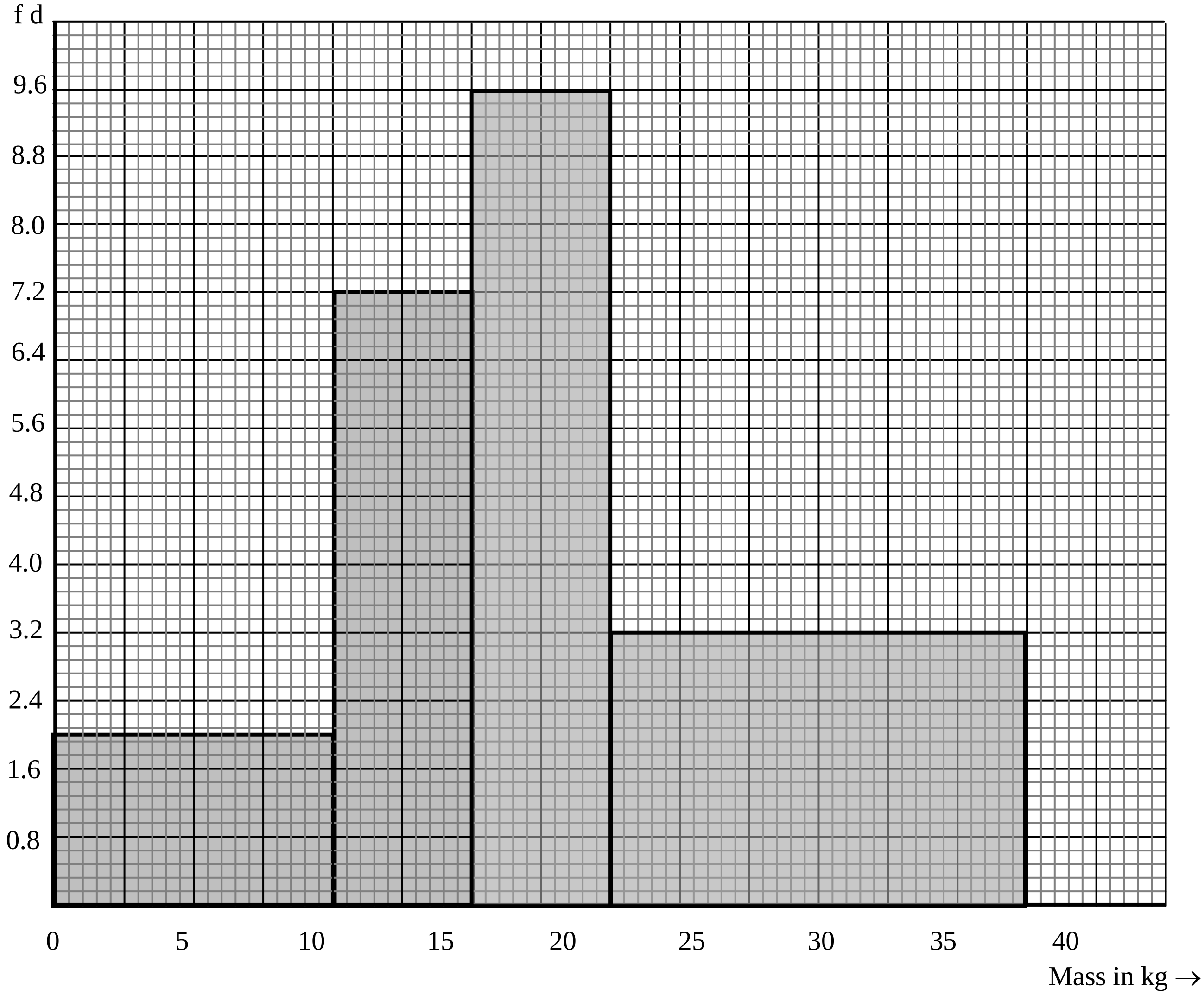
8. Answer the whole of this question on a sheet of graph paper.

152 passengers on an aircraft had their baggage weighed. The results are shown in the table.

Mass of baggage, M (kg)	$0 < M \leq 10$	$10 < M \leq 15$	$15 < M \leq 20$	$20 < M \leq 35$
Number of passengers	20	36	48	48

Using a scale of 2 cm to represents 5 kg draw a horizontal axis for $0 < M \leq 40$.

Using an area scale of 1 cm^2 to represent 2 passengers, draw a histogram for this data



9. (a) Modal score is 1. [It has the highest frequency]

(b) Least possible score is 3.

(c) Sum of the first 28 scores is 1 × 8 + 2 × 6 + 3 × 6 + 4 × 2 + 5 × 4 + 6 × 2 = 78. Now (78 + *a*) ÷ 30 = 3 or with *a* the sum of the last two scores. So 78 + *a* = 90 so *a* = 12. So the last two scores are both a six.

10. Mean length = (69 × 3 + 72 × 7 + 76 × 10 + 79.5 × 16 + 82 × 14) ÷ 50 = 77.82 = 78 cm

11. (a) Use formula: Frequency density × class width = frequency so for the column of 20→40: *f* × 3 × 20 = 30 so *f* = ½ [*f* is a constant value .] So for the column of 40→50 we get ½ × *x* × 10 = 20 so *x* = 4 cm. The column mentioned has a height of <u>4 cm</u>.

Another reasoning based in the idea that the density of the area in the histogram should be the same for each column: So the area for the column 20→40 is 3 × 20 = 60 this 60 represents a frequency of 30 so the 60 was halved. For the column 40→50 we get: fr dens × 10 × ½ = 20 so the frq dens must be <u>4 cm</u>.

Yet another reasoning would be: half the width of the column of 20→40; put the two bars on top of each other; they will reach a height of 6 cm. The column of the 40→50 is now $\frac{2}{3}$ of that height since 20 is $\frac{2}{3}$ of 30 and $\frac{2}{3}$ × 6 cm = <u>4 cm</u>.

(b) Modal interval is 20 < *m* ≤ 40.

Section 13 Probability

1. (a) P(The wind is not blowing) $\times$ P(She hits a straight drive) = $\frac{7}{10} \times \frac{8}{10} = 0.56$

(b) P(The wind is not blowing) $\times$ P(She hits a straight drive) +

P(The wind is blowing) $\times$ P(She hits a straight drive) = $\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{4}{10} = 0.56 + 0.12 = 0.68$

(c) $(1 - 0.68)(1 - 0.68) = 0.32^2 = 0.1024 = \frac{64}{625}$

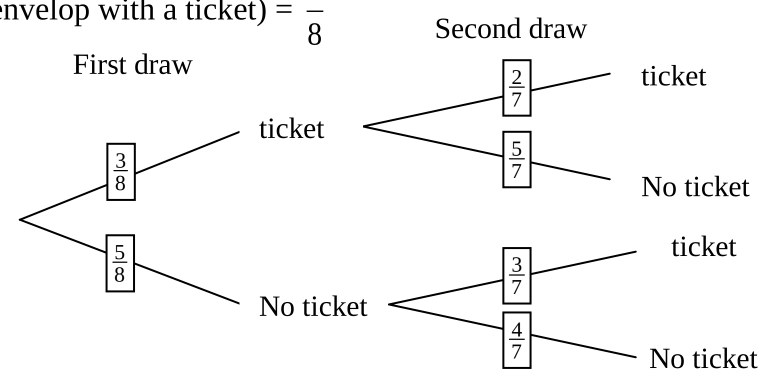
2. (a) P(both balls are red) = $\frac{5}{7} \times \frac{2}{6} = \frac{5}{21}$

(b) In the same way as in **(a)** P(both balls are white) = $\frac{2}{7} \times \frac{4}{6} = \frac{4}{21}$

so P(the balls are of different colours) = $1 - \frac{9}{21} = \frac{12}{21} = \frac{4}{7}$

3. (a) P(the first child picks an envelop with a ticket) = $\frac{3}{8}$

(b) Use a tree diagram:



So P(the second child picks an envelop with a ticket) = P(first pick a ticket and second pick a ticket) +

P(first pick not a ticket and the second pick is a ticket) = $\frac{3}{8} \times \frac{2}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{21}{56}$

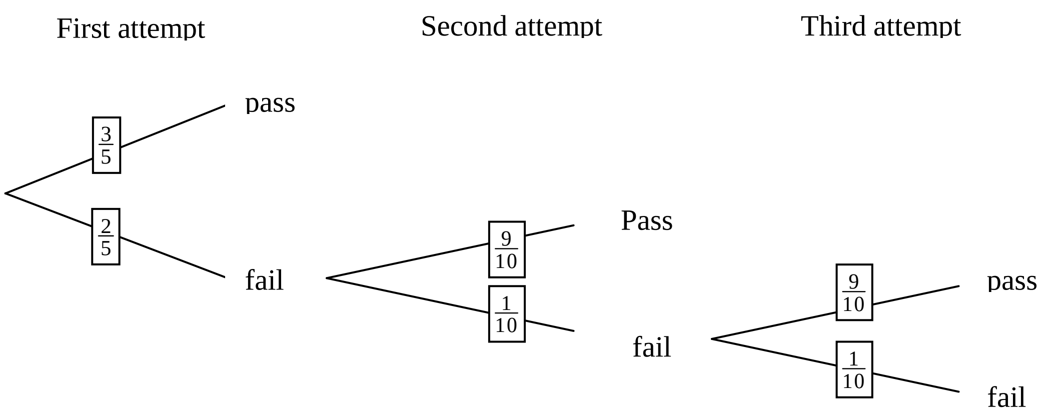
4. (a) P(he has never been to Namibia) = $\frac{6}{10} = \frac{3}{5}$

(b) P(at least one of the two has visited Namibia before) =

P(first person has visited N and the second not) + P(first person has not visited N and the second has visited)+

P(both have visited Namibia) **=** $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{3}{9} = \frac{24 + 24 + 12}{90} = \frac{60}{90} = \frac{2}{3}$

5. (a)

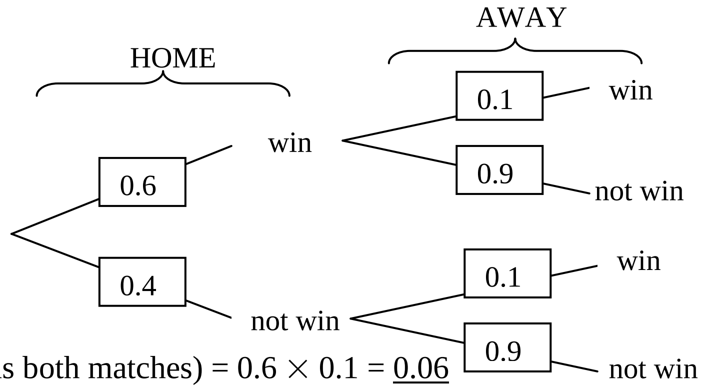


(b) P(no more than two attempts to pass the test) = P(pass at the first attempt)+ P(fail first, pass at second att.) = $0.6 + 0.4 \times 0.9 =$ <u>0.96</u>

(c) P(passes at first attempt) = 0.6

P(passes at second attempt) = $0.4 \times 0.9 = 0.36$
P(passes at third attempt) = $0.4 \times 0.1 \times 0.9 = 0.036$
P(passes at fourth attempt) = $0.4 \times (0.1)^2 \times 0.9 = 0.0036$
P(passes at the n^{th} attempt) = $0.36 \times 0.1^{n-2}$

6. (a)



(b) P(Rangers wins both matches) = 0.6×0.1 = <u>0.06</u>

(c) P(wins at least one of the matches) =
P(wins first, loses second match) + P(loses first, wins second match) + P(wins both) =
$0.6 \times 0.9 + 0.4 \times 0.1 + 0.6 \times 0.1 = 0.54 + 0.04 + 0.06 =$ <u>0.64</u>

7. (a) (i) Probability is 4 out of 30 = $\frac{2}{15}$ **(ii)** Probability is 11 out of 30 = $\frac{11}{30}$

(b) Probability = $\frac{19}{30} \times \frac{18}{29} = \frac{171}{435}$

8. (a) (i) $0.7 \times (1 - 0.18) = 0.7 \times 0.82 = 0.574$

(ii) Analyzing the whole situation:
① she swims but not in a bikini is: $0.7 \times (1 - 0.18) = 0.7 \times 0.82 = 0.574$
② she swims in a bikini: $0.7 \times 0.18 = 0.126$
③ she does not swim but appears in a bikini: $0.3 \times 0.18 = 0.054$
④ she does not swim and does not wear a bikini: $0.3 \times (1 - 0.18) = 0.246$.
Of course ① + ② + ③ + ④ = 1
The probability is ① + ③ + ④ = 0.574 + 0.054 + 0.246 = <u>0.874</u> [= 1 – P(swims in Bik.) = 1 – 0.126]

(b) Probability is $(1 - 0.18)^3 \times 0.18 = 0.09924624$.

9. (a) (i) $\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$ **(ii)** 60 = 50 + 10 or 60 = 10 + 50 so P(60 c) = $\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$

(b) 70 = 50 + 10 + 10 or 70 = 10 + 50 + 10 or 70 = 10 + 10 + 50.
So P(70 c) = $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{6+6+6}{60} = \frac{18}{60} = \frac{3}{10}$

10. (a) $\frac{3}{4} \times \frac{1}{5} \times \frac{2}{3} = \frac{1}{10}$

(b) P(has to stop at least once) = 1 − P(she does not have to stop) = $1 - \frac{1}{10} = \frac{9}{10}$

(c) P(has to stop exactly at two traffic lights) =P(green at TL①; other 2 red) + P(green at TL②; other 2 red)
+ P(green at TL③; other 2 red) = $\frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{5} \times \frac{1}{3} + \frac{1}{4} \times \frac{4}{5} \times \frac{2}{3} = \frac{12+1+8}{60} = \frac{21}{60} = \frac{7}{20}$.

Section 14 Algebra & Functions

1. (a) $fg(x) = f(g(x)) = f(2-4x) = \dfrac{6}{2(2-4x)+3} = \dfrac{6}{7-8x}$ with $x \neq \frac{7}{8}$

(b) $3 = \dfrac{6}{7-8x}$ multiply both sides by $(7-8x) \rightarrow 3(7-8x) = 6 \rightarrow 21 - 6 = 24x$ so that $x = \frac{5}{8}$

2. (a) $(2n)^{3+1} = (2n)^4 = 2^4 \times n^4 = 16\,n^4 = an^b$ this means $\underline{a = 16;\ b = 4.}$

$(3+1)^{2n} = 4^{2n} = 4^{2\times n} = (4^2)^n = 16^n$ so $\underline{c = 16}$

(b) $\underline{1 + 3 \times 2^n} > 1$ for all choices of n.

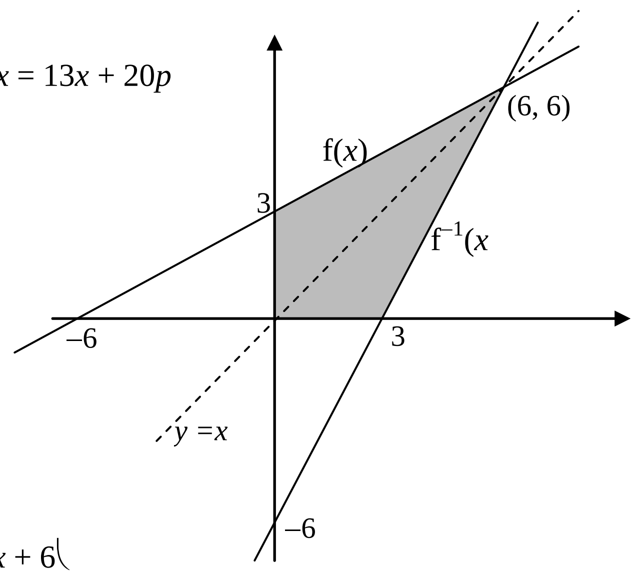
3. (a) The total number of books bought is: $4 + 3 + p = 7 + p$

(b) The total cost, in N$, of the books bought is $4x + 20p + 9x = 13x + 20p$

(c) $(4x + 20p + 9x) \div (7 + p) = \dfrac{13x+20p}{p+7}$.

4. (a) , (b)

If $f(x) = ½\,x + 3$ or $y = ½\,x + 3$ ….*
then $f^{-1}(x) = 2x - 6$. [You can find this by swopping x and y in * and make y subject of the equation.]
The relation of the graph of f and f^{-1} is a reflection in the line $y = x$.



5. (a) Divide $x^3 - 2x^2 - 5x + 6$ by $(x - 3)$:

```
            x² + x − 2
x − 3 ) x³ − 2x² − 5x + 6
      −  x³ − 3x²
              x² − 5x
            −  x² − 3x
                 − 2x + 6
               −  − 2x + 6
                        0
```

So area base is $x^2 + x - 2$

(b) $x^2 + x - 2 = (x+2)(x-1)$ so $\underline{k = 2 \text{ and } h = -1}$

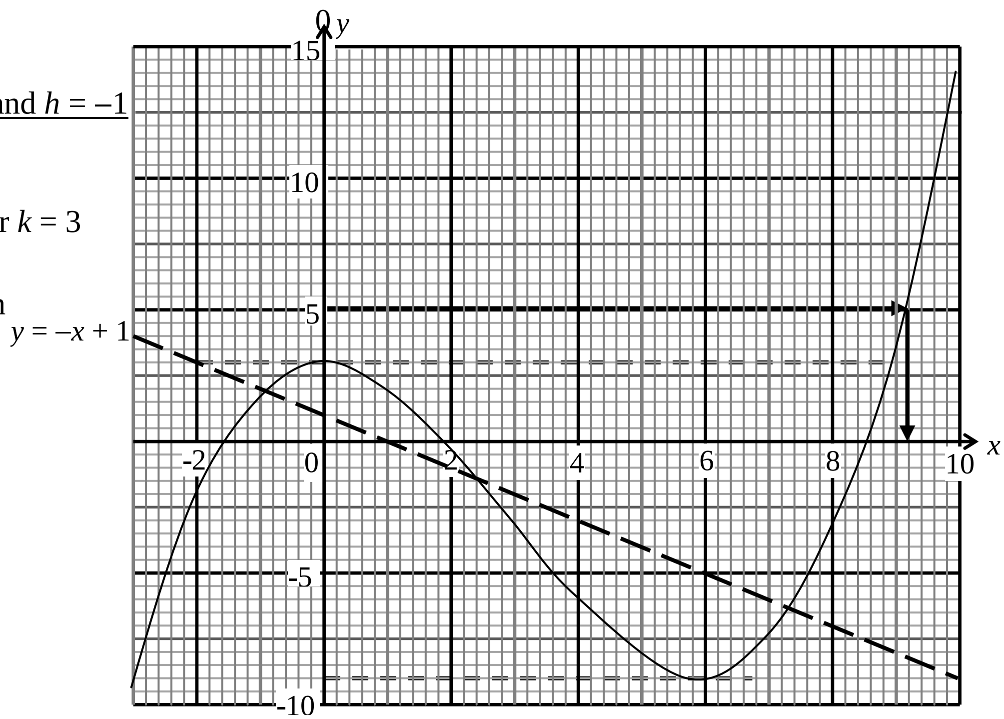
6. (a) from the graph: $\underline{f^{-1}(5) = 9.2}$

(b) see graph (the dotted lines) $k = -9$ or $k = 3$

(c) (i) $y = -x + 1$

(ii) Draw the graph of $y = -x + 1$ in the diagram (slanted line).

Conclusion: there are 3 solutions.



7. (a) $q = k/p \rightarrow$ **C**

(b) $q = k\,p^2 \rightarrow$ **B**

(c) $q = k\sqrt{p} \rightarrow$ **D**

(d) $q = k\,p \rightarrow$ **A**

8. (a) $\dfrac{3a^2-3}{3a^2} \times \dfrac{a^3-a^2}{a^2+a} = \dfrac{3(a-1)(a+1)}{3a^2} \times \dfrac{a^2(a-1)}{a(a+1)} = \dfrac{(a-1)^2}{a} = a-2+\dfrac{1}{a}$

(b)

$$\begin{array}{r}
x^2 - 2x - 3 \\
x+2 \,\overline{)\, x^3 - 7x - 6} \\
-\quad \underline{x^3 + 2x^2} \\
-2x^2 - 7x \\
-\quad \underline{-2x^2 - 4x} \\
-3x - 6 \\
-\quad \underline{-3x - 6} \\
0
\end{array}$$

So the cost of one book is N$($x^2 - 2x - 3$).

9. $a + b = 6 \rightarrow (a + b)^2 = 36$ or $a^2 + 2ab + b^2 = 36$ then $a^2 + b^2 = a^2 + 2ab + b^2 - 2ab = 36 - 14 = \underline{22}$

Section 15 Coordinate geometry

1. (a) The gradient is $\dfrac{6-(-4)}{-3-9} = \dfrac{10}{-12} = -\frac{5}{6}$ so the equation becomes: $y = -\frac{5}{6}x + c$

Point $A(-3, 6)$ is on the line: $6 = -\frac{5}{6}\times(-3) + c$ so $c = 3.5$ so the equation is: $y = -\frac{5}{6}x + 3.5$

(b) Midpoint of AB is $M(½[9 - 3]\,,\, ½[6 - 4])$ or $M(3, 1)$; the gradient of the perpendicular is 1.2. [$-\frac{5}{6} \times \frac{6}{5} = -1$]

$M(3, 1)$ is on the line $y = 1.2\,x + c \rightarrow 1 = 3.6 + c$ so $c = -2.6$. The requested equation is $\underline{y = 1.2\,x - 2.6.}$

Of course this is the same as $y = 1\frac{1}{5}x - 2\frac{3}{5}$.

(c) $13 = 1.2 \times 13 - 2.6$ or $13 = 13$ so $C(13, 13)$ is on the locus

(d) $AC = \sqrt{[13-(-3)]^2 + [13-6]^2} = \sqrt{256 + 49} = \sqrt{305}$ (= 17.5) [The 17.5 can be left out; √305 is an exact answer, that does not need to be rounded off to three decimals. If you make a mistake with rounding off, you may loose marks]

2. $x - y + 1 = 0 \rightarrow x = y - 1$ substitute in $x^2 + y^2 = 9 \rightarrow (y - 1)^2 + y^2 = 9$ or $2y^2 - 2y - 8 = 0$ so that

$y^2 - y - 4 = 0 \rightarrow y = \dfrac{1 \pm \sqrt{1-4\times1\times(-4)}}{2} = \dfrac{1 \pm \sqrt{17}}{2}$.

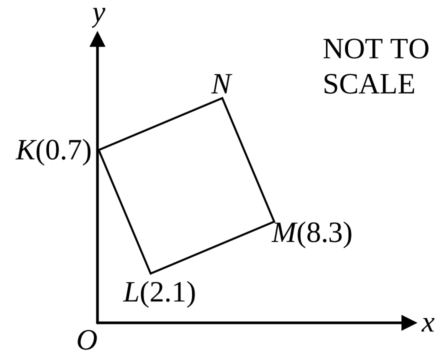
One point is: $y = 0.5 + ½\sqrt{17}$ which gives $x = -0.5 + ½\sqrt{17}$ resulting in: (1.6, 2.6)

The other point is $y = 0.5 - ½\sqrt{17}$ which gives $x = -0.5 - ½\sqrt{17}$ resulting in: (–2.6, –1.6)

3. (a) (i) Can be read from the graph: $y = -3x + 7$.

More algebraic: gradient is $\frac{1-7}{2-0} = \frac{-6}{2} = -3$

So the equation is $y = -3x + c$ and $c = 7$ because the line KL cuts the y-axis in 7 so KL: $y = -3x + 7$



(ii) The line is parallel to KL [same gradient!] and passes through the midpoint of LM.

Midpoint LM is $\left(\frac{2+8}{2}, \frac{1+3}{2}\right)$ = (5, 2) so the equation is $y = -3x + c$; (5, 2) on it so $2 = -15 + c$ or

$c = 17$ so the perpendicular bisector of LM has equation <u>$y = -3x + 17$</u>

(b) Length $KL = \sqrt{(2-0)^2 + (1-7)^2} = \sqrt{4+36} = \sqrt{40}$ so the area of the square is <u>40 square units.</u>

4. (a) AC has gradient $\frac{6-(-1)}{5-(-2)} = \frac{7}{7} = 1$ so the equation of AC becomes $y = x + c$; (5, 6) on the line so

$6 = 5 + c$ then $c = 1$. So the equation of AC is $y = x + 1$

(b) M is midpoint of AC so $M\left(\frac{-2+5}{2}, \frac{-1+6}{2}\right)$ = <u>$M(1.5, 2.5)$</u>

(c) BD has gradient -1 so BD: $y = -x + c$. $M(1.5, 2.5)$ on the line so $2½ = -1½ + c \rightarrow c = 4$. BD: <u>$y = -x + 4$</u>

(d) From B to M the x values decreases by 0.5 and the y coordinate increases by 0.5 so $D(1, 3)$

(e) $BC = \sqrt{(6-2)^2 + (5-2)^2} = \sqrt{16+9} = 5$ so the perimeter is <u>20 units.</u>

5. (a) $BA = BC$ so $\sqrt{[6-(-2)]^2 + (9-3)^2} = \sqrt{[(-2)-h]^2 + [3-(-5)]^2}$ rewrite both square roots:

$\sqrt{64+36} = \sqrt{h^2 + 4h + 4 + 64}$ square both sides:

$100 = h^2 + 4h + 68$ write in the standard form:

$h^2 + 4h - 32 = 0$ or $(h + 8)(h - 4) = 0$ so <u>$h = 4$</u> [$h = -8$ is rejected: not positive.]

(b) gradient AB is $\frac{9-3}{6-(-2)} = \frac{6}{8} = \frac{3}{4}$

gradient BC is $\frac{-5-3}{4-(-2)} = \frac{-8}{6} = \frac{-4}{3}$

And $\frac{3}{4} \times \frac{-4}{3} = -1$ so AB and BC are perpendicular.

(c) Midpoint AB is (½[−2 + 6], ½[9 + 3]) = (2, 6) ; MP is parallel to BC so the equation is $y = -1\frac{1}{3}x + c$.

Point (2, 6) is on this line: so $6 = -2\frac{2}{3} + c$ so that $c = 8\frac{2}{3}$; the equation is now: $y = -1\frac{1}{3}x + 8\frac{2}{3}$.

6. (a) $4x - y = 9 \rightarrow y = 4x - 9$ substitute in $y = 3 + 13x - 3x^2$: $4x - 9 = 3 + 13x - 3x^2$ or $3x^2 - 9x - 12 = 0$

Divide by 3: $x^2 - 3x - 4 = 0$ or $(x - 4)(x + 1) = 0$ so $x = -1$ or $x = 4$

(b) $P(-1, -13)$ and $Q(4, 7)$ so $PQ = \sqrt{5^2 + 20^2} = \sqrt{425} = 20.6$

(c) Midpoint PQ is $M(1.5, -3)$ the gradient of PQ is $[7-(-13)] \div [4-(-1)] = 20 \div 5 = 4$ so the perpendicular has gradient -0.25 so the equation is $y = -0.25x + c$ or $-3 = -0.25 \times 1.5 + c$

$c = -3 + 0.375 = -2.625$ so the equation is $y = -0.25x - 2.625$ this is the same as $y = -\frac{1}{4}x - 2\frac{5}{8}$

7. (a) $y = -x + 5$ [If you want to write something more, you can write $B(2,3)$ on $y = -x + c$ or $3 = -2 + c$ so $c = 5$]

(b) Solve simultaneously $y = \frac{1}{4}x$ and $y = -x + 5$ or $\frac{1}{4}x = -x + 5$ multiply with 4: $5x = 20 \rightarrow x = 4$ then $y = 1$

so <u>$A(4,1)$</u>

(c) Midpoint of OB is (½[2 + 0] , ½[3 + 0]) which is <u>(1, 1.5)</u>

(d) Perpendicular bisector of OB has gradient $-\frac{2}{3}$ [Product with gradient of OB = –1!]

and the line passes through (1, 1.5) so substitute in: $y = -\frac{2}{3}x + c \rightarrow 1.5 = -\frac{2}{3} + c$ so $c = 2\frac{1}{6}$

So the equation is $y = -\frac{2}{3}x + 2\frac{1}{6}$.

8. (a) Midpoint AC is <u>(6, 1)</u>

(b) Area ΔABC = ½ × base × height = ½ × 6 × 8 = <u>24 sq units</u>

(c) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{6-(-2)} = -½$. So the equation is $y = -½x + c$. B is on the line so $2 = -½ \times (-2) + c$.

this means $c = 1$. The equation is <u>$y = -½x + 1$.</u>

(d) The angle the line makes with the positive x-axis is $\tan^{-1}(-0.5) = -26.6^\circ$ so θ = 180 – 26.6 = <u>153.4°</u>

(e) $y + 2x = 8$ so the gradient is –2. So the equation is $y = -2x + c$. $A(6, 4)$ is on the line → $4 = -12 + c$ so $c = 16$. The equation is <u>$y = -2x + 16$.</u>

9. (a) (i) Let the line be $y = mx + c$ then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-4}{9-1} = -\frac{6}{8} = -\frac{3}{4}$. $A(1, 4)$ is on the line so $4 = -\frac{3}{4} \times 1 + c$

So $c = 4\frac{3}{4}$ the equation is $y = -\frac{3}{4}x + 4\frac{3}{4}$ multiply with 4: <u>$3x + 4y - 19 = 0$</u>

(ii) BD is perpendicular to AC so the gradient of DB is $\frac{4}{3}$ [Product of the gradients is −1]

M is the midpoint of DB so $M\left(\dfrac{1+9}{2}, \dfrac{4-2}{2}\right)$ or $M(5, 1)$ and M is on the line BD.

Let the equation be $y = mx + c$ or $y = \frac{4}{3}x + c$; substitute M: $1 = \frac{4}{3} \times 5 + c \rightarrow c = -5\frac{2}{3}$

Requested equation is $y = \frac{4}{3}x - 5\frac{2}{3}$ or multiply with 3: $-4x + 3y + 17 = 0$ or $4x - 3y - 17 = 0$

(b) $D(AC) = \sqrt{([9-1)^2 + (4+2)^2]} = \sqrt{(64 + 36)} =$ <u>10</u>.

<u>Section 16 Geometry including circle geometry</u>

1. (a) $\angle DAB$ = 180 – 80 = 100° then $\angle ABD$ = ½(180° – 100°) = <u>40°.</u>

(b) $\angle BDY$ = 40° + 100° = <u>140°</u>

2. (a) angle OCB = 90° – x

(b) angle CAB = angle CDB but angle $CDB = x$ [angle BCQ and angle CDB are both subtended by arc BC.
so angle $CAB = x$.

(c) angle DOC = <u>$2y$</u>

(d) angle ODC = ½(180° – 2y) = 90° – y [ΔDOC is an isosceles triangle]

3. (a) (i) Angle ACT = 90° [TC is tangent and AC is diameter.] so x = 90° – 35° = 55°

(ii) Angle ABC = 90° [Angle in a semi circle] so y = 180° – 90° – 55° = 35°

(b) $z = x$ = 55° [angle ADB and angle ACB are both subtended by minor arc AB]

4. (a) x = 2 × 38° = <u>76°</u>

(b) y = 90 – 38 = <u>52°</u>

(c) z = <u>38°</u>

5. (a) Angle BDA = x° as well both subtended by chord AB.

(b) Angle CDA = 180° – y°, yet the sum of ABC and CDA = 180° [$ABCD$ is cyclic quadrilateral]

So angle ABC = 180º – (180º – yº) = <u>yº</u>

(c) angle <u>BAC = 180º – xº – yº</u>

(d) $\angle BAG = x$º so angle AGB + x = angle ABC = yº so <u>angle AGB = yº – xº</u>

6. (a) Angle CBE = 90º then angle ABC = 90º as well. This means AC is the diameter of the circle.

(b) (i) angle BCD = 180 – xº [$ABCD$ is a cyclic quadrilateral]

(ii) angle BEC = 90º – xº [angle DCB + angle BCE = 180º (straight line) and angles in triangle BCE add up to 180º]

(c) They are similar because they have one angle in common (angle E) and angle EBC = angle ADE = 90º.

(d) Area triangle ADE : area triangle CBE = 16 : 9.

7. (a) (i) angle ODC = ½(180 – 76) = <u>52°.</u>

(ii) angle ABC = 180 – 52 = <u>128°</u> [Opposite angles in a cyclic quadrilateral add up to 180°]

(iii) first angle OCD = 52° [see **(i)**] Then angle OCA = ½[180 − (180 – 76)] = 38°.
Quadrilateral $ABCD$ is cyclic so angle DCB = 180 – 76 = 104°. So angle ACB = 104 – 38 – 52 = <u>14°.</u>

<u>Section 17 Equations, substitutions & changing the subject of a formula</u>

1. (a) ΔABC is half of an equilateral triangle so $2 \times (x^2 - 4) = 3x + 1$ or $2x^2 - 8 = 3x + 1$ write in standard form: $2x^2 - 3x = 9$

(b) $2x^2 - 3x = 9 \rightarrow \xrightarrow{\div 2} x^2 - 1.5x = 4.5$ or $x^2 - 1.5x + \frac{9}{16} = 4.5 + \frac{9}{16}$ or $(x - \frac{3}{4})^2 = 5\frac{1}{16}$ take square root from both sides: $x - \frac{3}{4} = \pm\sqrt{\frac{81}{16}}$ so that $x = \frac{3}{4} \pm \frac{9}{4}$ which gives the values <u>$x = 3$ or $x = -1½$</u>

(c) $AB = 3^2 - 4 =$ <u>5</u> [$x = -1½$ cannot be used because AC will be a negative distance for $x = -1½$]

2. (a) $x = \dfrac{3y+5}{2y} = \dfrac{32}{18} = \dfrac{16}{9} = 1\frac{7}{9}$

(b) $x = \dfrac{3y+5}{2y} \xrightarrow{\text{multiply with } 2y} 2xy = 3y + 5$ or $2xy - 3y = 5$ or $y(2x - 3) = 5$ so $y = \dfrac{5}{2x-3}$

3. (a) $\dfrac{3}{(x+1)(x+3)} - \dfrac{2}{(x+3)} = \dfrac{3}{(x+1)(x+3)} - \dfrac{2(x+1)}{(x+1)(x+3)} = \dfrac{3-2x-2}{(x+1)(x+3)} = \dfrac{-2x+1}{(x+1)(x+3)}$

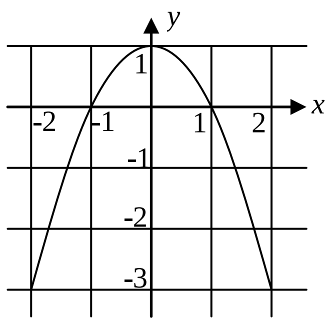
(b) $\dfrac{3}{(x+1)(x+3)} - \dfrac{2}{(x+3)} = 0$ is the same as $\dfrac{-2x+1}{(x+1)(x+3)} = 0$ which means $-2x + 1 = 0$ so <u>$x = ½$</u>

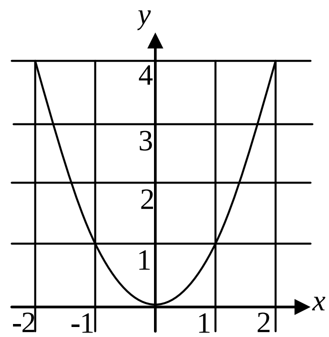
4. (a) $y = \dfrac{7}{\sqrt{2.53 \times 10^{-8}}} - 70 = 43938.622942335 =$ <u>4.39×10^4</u>

(b) $y = \dfrac{7}{\sqrt{x}} - 70 \xrightarrow{\text{isolate } x \text{ first}} y + 70 = \dfrac{7}{\sqrt{x}} \xrightarrow{\text{square both sides}} ((y+70)^2 = \dfrac{49}{x}$ so $x = \dfrac{49}{(y+70)^2}$

Remark: you may work out the square in the denominator but it is not necessary, however if you make a mistake in working out $(y + 70)^2$ you will lose marks.

5. (a) (i) take $p = -1$ and $r = 1$. **(ii)**





(b) (i) Method I [completing the square]

$2x^2 - 3x - 4 = 2[x^2 - 1.5x\,] - 4 = 2[x^2 - 1.5x + \frac{9}{16} - \frac{9}{16}] - 4 = 2[(x - \frac{3}{4})^2 - \frac{9}{16}] - 4 = 2(x - \frac{3}{4})^2 - 5\frac{1}{8}$

Method II [See $2x^2 - 3x - 4 = 2(x + a)^2 + b$ as an identity]

$2x^2 - 3x - 4 = 2(x + a)^2 + b$

$2x^2 \underline{- 3x} - 4 = 2x^2 \underline{+ 4ax} + 2a^2 + b$ from the two underlined terms we conclude:

$-3 = 4a$ so $a = -\frac{3}{4}$.

Check now the constant terms of both sides: $-4 = 2a^2 + b$ or $-4 = 2 \times \frac{9}{16} + b$ so that $b = -5\frac{1}{8}$

The requested expression is now: $2(x - \frac{3}{4})^2 - 5\frac{1}{8}$.

(ii) The coordinates of the stationary point of the curve $y = 2x^2 - 3x - 4$ are $(\frac{3}{4}, -5\frac{1}{8})$

6. $\frac{x+1}{x-1} - \frac{x+2}{x} = 2$ or $\frac{x+1}{x-1} - \frac{x+2}{x} - 2 = 0$ give all terms the same denominator of $x(x - 1)$

$\frac{x(x+1)}{x(x-1)} - \frac{(x+2)(x-1)}{x(x-1)} - \frac{2x(x-1)}{x(x-1)} = 0$ or $\frac{x^2+x}{x(x-1)} - \frac{x^2+x-2}{x(x-1)} - \frac{2x^2-2x}{x(x-1)} = 0$ or $\frac{-2x^2+2x+2}{x(x-1)} = 0$

This leads to the quadratic equation $x^2 - x - 1 = 0$; the solutions are: $x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{5}$ so $\underline{x = 1.62}$ or $\underline{x = -0.62}$

7. (b) Use cross multiplication: $T(V^2 + 4) = 5$ so $TV^2 = 5 - 4T$ so $V = \sqrt{\frac{5-4T}{T}}$

8. (a) $T - 2 = \frac{4a}{3+4b^2}$ multiply both sides with $(3 + 4b^2)$: $(3 + 4b^2)(T - 2) = 4a$

Divide both sides by $(T - 2)$: $3 + 4b^2 = \frac{4a}{T-2}$

Subtract 3 from both sides: $4b^2 = \frac{4a}{T-2} - 3$

Take the square root from both sides: $2b = \sqrt{\frac{4a}{T-2} - 3}$

Multiply both sides with ½: $b = \frac{1}{2}\sqrt{\frac{4a}{T-2} - 3}$

(b) Express $y = -\frac{1}{3}x^2 - 4x + 2$ in the form $y = a(x + p)^2 + q$.

Work out $a(x + p)^2 + q = ax^2 + 2axp + ap^2 + q \equiv -\frac{1}{3}x^2 - 4x + 2$ so $\underline{a = -\frac{1}{3}}$ and $2ap = -4$ or $2 \times (-\frac{1}{3})\,p = -4 \rightarrow \underline{p = 6}$;

and $ap^2 + q = 2$ or $(-\frac{1}{3}) \times 36 + q = 2 \rightarrow \underline{q = 14}$.

Section 18 Construction; locus and symmetry.

1 . (a) (i) diagram 5 **(ii)** diagram 4 **(iii)** diagram 2

(b) $x = ½ \times 45° = \underline{22½°}$

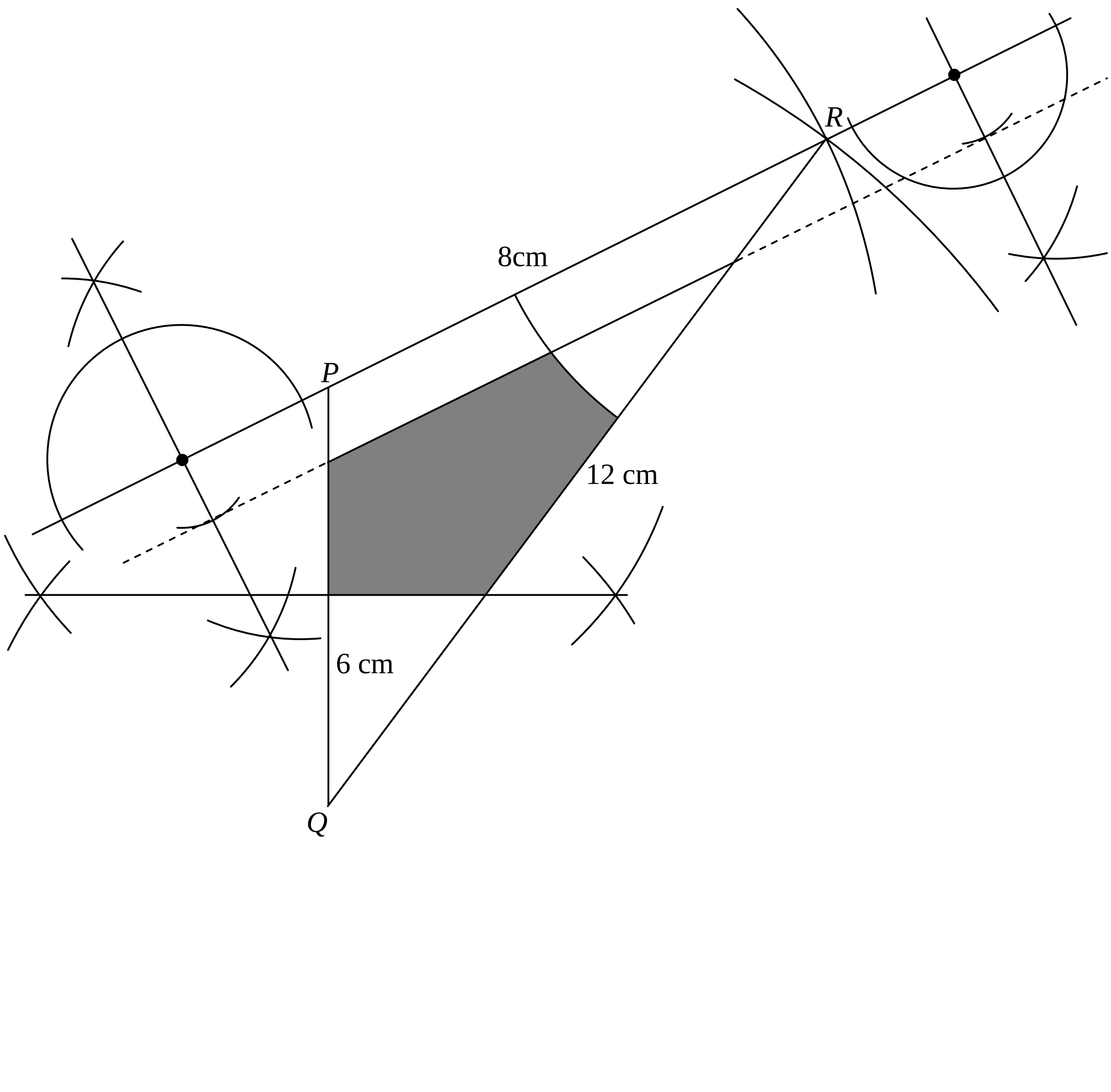
(c) (i) Area = $½\, bh = ½ \times 3 \times 6 = 9$ cm^2

(ii) Height of the shade3d triangle is 3 cm so area = $½\, bh = ½ \times 3 \times 3 = 4.5$ cm^2

(iii) Use cosine to calculate the length of the angular bisector: $\cos 22½ = \dfrac{6}{\text{length bisector}}$

S the length of the bisector is 6.49 cm. Area shaded part is $½ \times (\text{bisector}) \times 6\sqrt{2}\ \sin 22½° = \underline{10.5 \text{ cm}^2}$

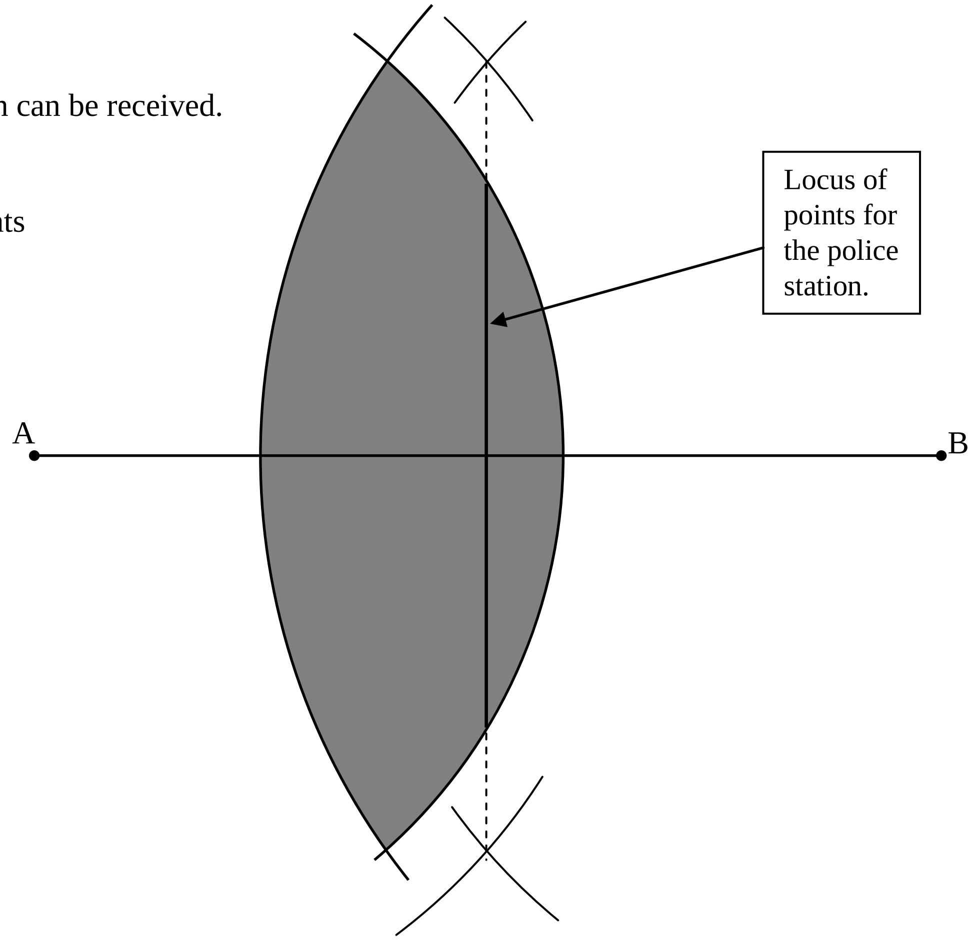
2.



3. (a)

(b) Shaded part both station can be received.

(c) The points on the thick line inside the shaded area is the locus of points



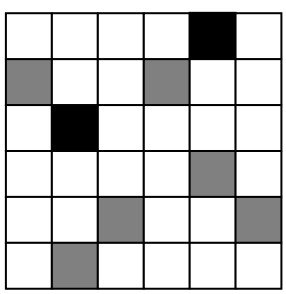
4. (a) E, T, O, H and A

(b) O, S and H

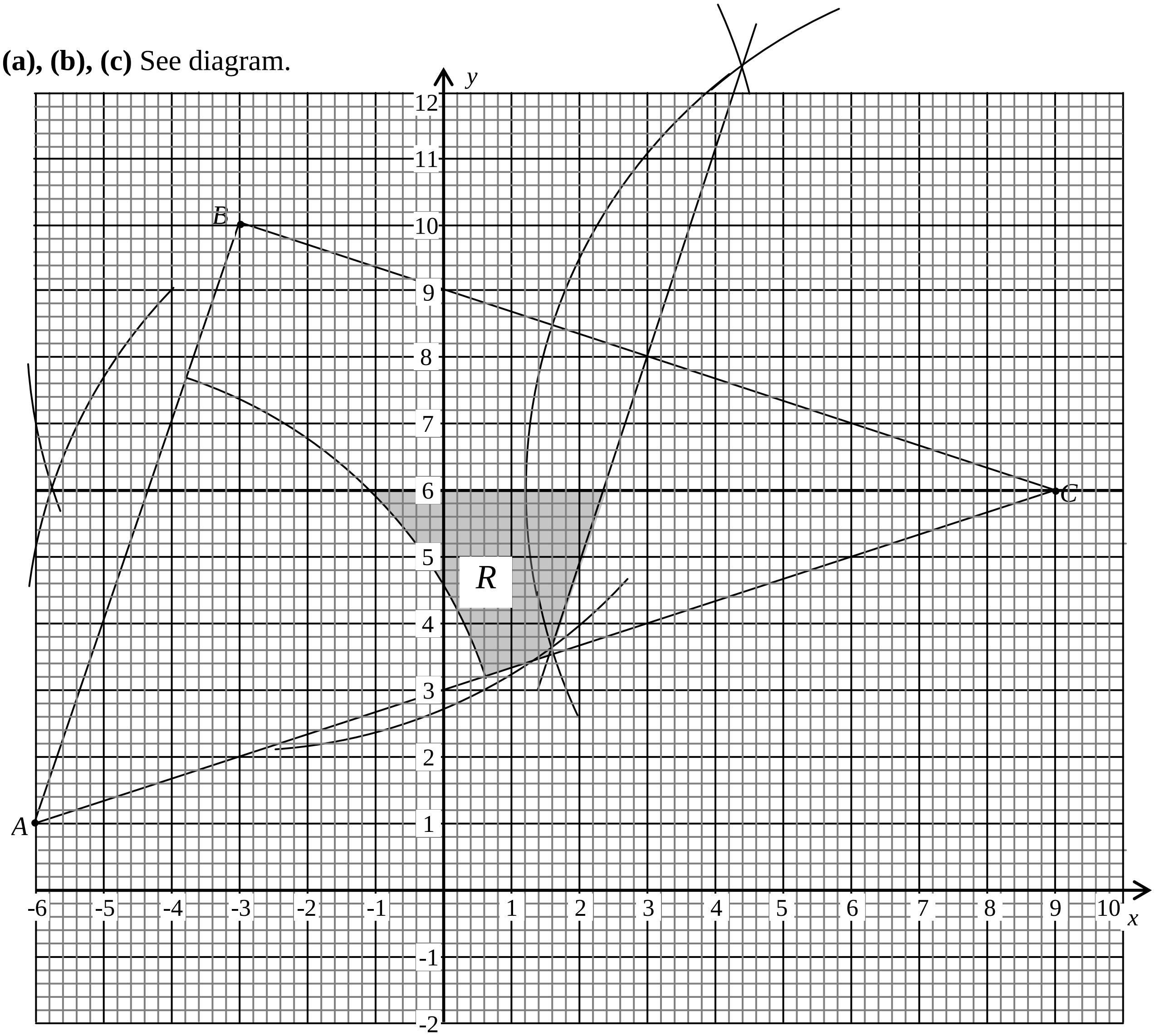
(c) 162 ÷ 72 = 2.25 hours.

(d) 161.5 ÷ 87.5 = 1.85 hours.

5.



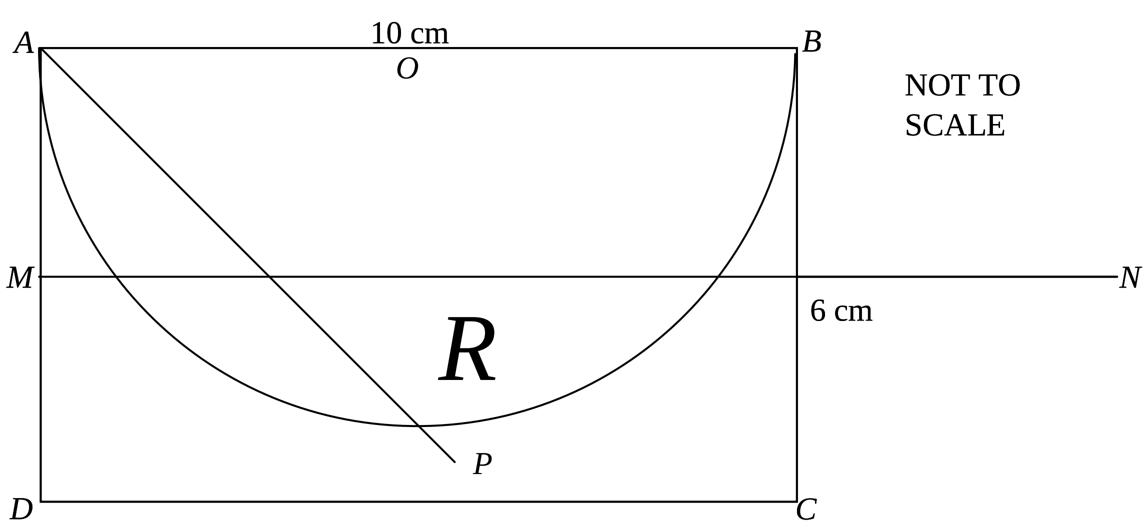
6. (a), (b), (c) See diagram.



7. (a) I and O

(b) I, N and O. [The N is not having perfect rotational symmetry]

8.



Section 19 2D vectors.

1. (a) (i) $\boldsymbol{DC} = \boldsymbol{DO} + \boldsymbol{OC} = \underline{-\mathbf{d} + \mathbf{c}}$

(ii) $\boldsymbol{OE} = \frac{1}{2}\boldsymbol{OC} + \boldsymbol{CD} = \frac{1}{2}\mathbf{c} + \mathbf{d} - \mathbf{c} = \underline{\mathbf{d} - \frac{1}{2}\mathbf{c}}$

(iii) $\boldsymbol{OB} = \boldsymbol{OC} + \boldsymbol{CB} = \boldsymbol{c} + \boldsymbol{EO} = \boldsymbol{c} + \frac{1}{2}\mathbf{c} - \mathbf{d} = \underline{1\frac{1}{2}\boldsymbol{c} - \boldsymbol{d}}$

(b) (i) Sum of interior, angles is $4 \times 180° = 720°$ [The hexagon can be covered by 4 triangles]

So angle $ABC = 720° \div 6 = \underline{120°}$

(ii) The area of $\triangle ABC = \frac{1}{2} AB\,BC \sin 120° = \frac{1}{2} \times 10^2 \sin 120° = \underline{43\text{ cm}^2}$

(iii) $AC^2 = AB^2 + BC^2 - 2AB\,BC \cos 120° = 100 + 100 + 200 \cos 120 = 300$ so $\underline{AC = 17.3\text{ cm}}$.

2. (a) $\overrightarrow{BC} = \boldsymbol{BA} + \boldsymbol{AD} + \boldsymbol{DC} = -\mathbf{a} + \mathbf{b} + 4\mathbf{a} = \underline{3\mathbf{a} + \mathbf{b}}$

(b) $\overrightarrow{BM} = \frac{1}{2}\boldsymbol{BC} = \frac{1}{2}(3\mathbf{a} + \mathbf{b}) = \underline{1\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}}$

(c) $\overrightarrow{AN} = \frac{3}{7}\boldsymbol{AM} = \frac{3}{7}(\boldsymbol{AB} + \boldsymbol{BM}) = \frac{3}{7}(\mathbf{a} + 1\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}) = 1\frac{1}{14}\mathbf{a} + \frac{3}{14}\mathbf{b}$

3. (a) (i) reflection in the line OQ

[Sequence of the letters matters, so rotation 60º clockwise about O is incorrect]

(ii) Rotation of 180º about O.

(b) Scale factor is 2.

(c) (i) $\overrightarrow{TR} = -\boldsymbol{a} + \boldsymbol{b}$ **(ii)** $\overrightarrow{OQ} = \boldsymbol{b}$ **(iii)** $\overrightarrow{RQ} = \boldsymbol{a} + \boldsymbol{b}$

4. (a) $\overrightarrow{ZX} = \boldsymbol{ZO} + \boldsymbol{OX} = -\boldsymbol{b} + \boldsymbol{a}$

(b) $\overrightarrow{ZL} = \frac{2}{3}\mathbf{ZX} = -\frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{a}$

(c) $\overrightarrow{YL} = \mathbf{YZ} + \mathbf{ZL} = -\boldsymbol{a} + (-\frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{a}) = -\frac{1}{3}\boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$

(d) Position vector of L is $\boldsymbol{OL} = \boldsymbol{OZ} + \boldsymbol{ZL} = \boldsymbol{b} + -\frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{a} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

5. (a) $\overrightarrow{PQ} = \boldsymbol{PM} + \boldsymbol{MQ} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

(b) $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$

6. (a) (i) $\underline{DB} = \underline{DC} + \underline{CB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(ii) $|\overrightarrow{CD}| = \sqrt{[(-5)^2 + 1^2]} = \sqrt{26} \approx \underline{5.10}$

(b) (i) Trapezium

(ii) $\mathbf{DA} = \mathbf{DC} + \mathbf{CB} + \mathbf{BA} = 3\mathbf{p} - \mathbf{q} - 2\mathbf{p} = \underline{\mathbf{p} - \mathbf{q}}$

7. (a) $\text{tang}^{-1}(2 \div 1) = 63°$.

(b) $\underline{k = 50}$ because $2 \times 50 = 100$.

(c) (50, 100)

(d) Vector is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ [Can be seen by using the scalar product.]

8. (a) $\boldsymbol{AC} = \boldsymbol{AB} + \boldsymbol{BC} = 2\mathbf{a} + 3\mathbf{b}$ **(b)** $\boldsymbol{CD} = \frac{1}{2}(2\mathbf{a} + 3\mathbf{b}) = \mathbf{a} + 1\frac{1}{2}\,\mathbf{b}$

(c) $\boldsymbol{CE} = \mathbf{b}$ **(d)** $\boldsymbol{ED} = \boldsymbol{EC} + \boldsymbol{CD} = -\mathbf{b} + \mathbf{a} + 1\frac{1}{2}\,\mathbf{b} = \mathbf{a} + \frac{1}{2}\mathbf{b}$

9. (a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ -9 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

(b) Take the scalar product: $\overrightarrow{OB} \cdot \overrightarrow{OA} = \dfrac{a_1b_1 + a_2b_2}{|\overrightarrow{OB}| \times |\overrightarrow{OA}|} = \dfrac{6\times3+3\times(-6)}{\sqrt{45}\times\sqrt{45}} = 0$ so scalar product is zero so

$\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$.

(c) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ so $P(10, 5)$.

Section 20 Logs and exponents.

1. (a) $5^a = 25(5^{3b}) \rightarrow 5^a = 5^2 \times (5^{3b})$ or $5^a = 5^{2\,+\,3b}$ so that $a = 2 + 3b$ or $a - 3b = 2$ …….①

$\log(3a + 6) - \log(b + 1) = 1$ using the second law: $\log a - \log b = \log \frac{a}{b}$

$\log \dfrac{3a+6}{b+1} = \log 10$ this means $\dfrac{3a+6}{b+1} = 10$ so that $3a + 6 = 10(b + 1) \rightarrow 3a - 10b = 4$ …….②

(b) 3 × ① gives: $3a - 9b = 6$

②: $-\quad 3a - 10b = 4$

$0 \;+ \underline{b \;=\; 2}$ substitute in ①: $a - 6 = 2$ so $\underline{a = 8}$

2. $\dfrac{8^{n+1}}{2^{n-2} \times 2^{n+4}} = 2^k \xrightarrow{8\,=\,2^3} \dfrac{2^{3(n+1)}}{2^{n-2} \times 2^{n+4}} = 2^k$ so $2^{3n+3-(n-2)-(n+4)} = 2^k$

Or $3n + 3 - n + 2 - n - 4 = k$. This means $\underline{k = n + 1}$

3. $3(2^x) = 5 \rightarrow 2^x = \frac{5}{3}$ take logarithm from both sides: $\log 2^x = \log(5 \div 3)$ so that $x \log 2 = \log(5 \div 3)$

$x = \dfrac{\log(5\div3)}{\log 2} = \underline{0.737}$

4. (a) $5^{x+y} = 5^x \times 5^y = 4 \times 6 = 24$.

(b) $\frac{1}{2} \times 2^{20} = \frac{1}{2} \times 2 \times 2^{19} = 2^{19} = 524288 = 5.24288 \times 10^5$

(c) $x(x-3) - 10 = 0 \rightarrow x^2 - 3x - 10 = 0$ or $x^2 - 2 \times \frac{3}{2}x + \frac{9}{4} - \frac{9}{4} - 10 = 0$ so that $(x - \frac{3}{2})^2 - 12\frac{1}{4} = 0$

$(x - \frac{3}{2})^2 = 12\frac{1}{4}$ Take square root from both sides: $x - \frac{3}{2} = \pm\sqrt{\frac{49}{4}}$ this means $\underline{x = 5}$ or $\underline{x = -2}$

5. (a) $\dfrac{6a^2-6}{6a} \times \dfrac{a^3-a^2}{a^2-2a+1} = \dfrac{6(a+1)(a-1)}{6a} \times \dfrac{a^2(a-1)}{(a-1)^2} = a(a+1)$ or $a^2 + a$

(b) $\dfrac{5^{2n+1}\times 3^{\,2n-3}}{15^{\,2n}} = \dfrac{5^{2n+1}\times 3^{\,2n-3}}{3^{2n}\times 5^{2n}} = 5^{2n+1-2n}\times 3^{2n-3-2n} = 5\times 3^{-3} = \dfrac{5}{27}$

6. (a) 1kilobyte = $2^{10}\times 8 = 2^{10}\times 2^{3} = 2^{13}$ bits so $\underline{x = 13}$

(b) 4 kilobytes = $4\times 2^{13} = 2^{15}$ bits so $\underline{y = 15}$